SYNTHESIS OF EIGHT-LINK MECHANISMS

FOR A VARIETY OF MOTION

PROGRAMS

By

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CHAPTER I

INTRODUCTION

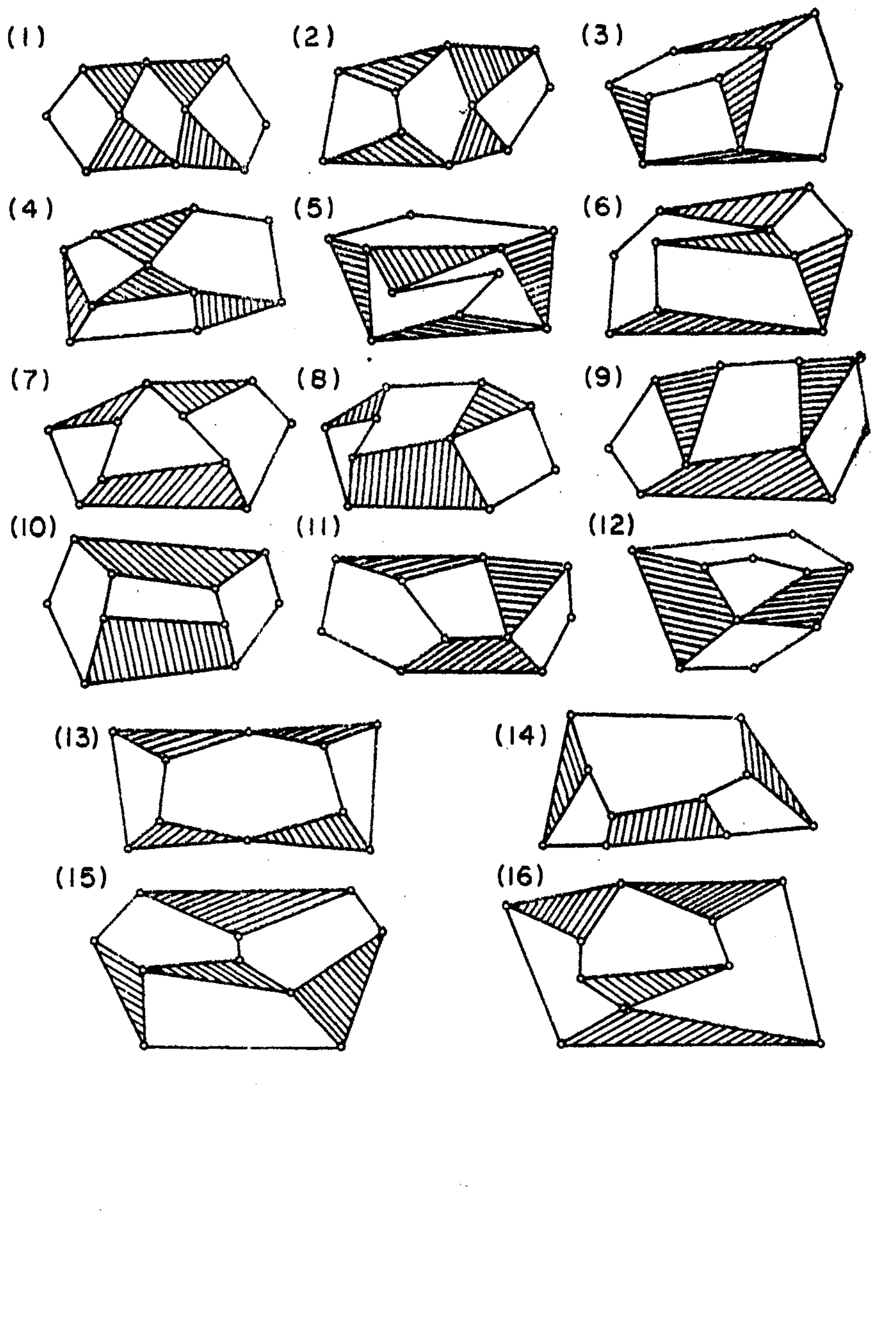
Recently, Soni and Harrisberger [1]* surveyed the state-of-the-art of mechanisms science and revealed the existence of nearly twelve thousand publications of a scholarly level. A detailed examination of these publications [2, 3] shows that, in recent years, there has been a considerable interest in synthesis of planar multi-loop eight-link mechanisms [4-49].

Using condensed molecule technique and graph theory, Franke [4] and Crossley [5] showed that there are only sixteen kinematic chains (shown in Table I) that can be synthesized from eight links and ten joints to have one degree of freedom. Historically, the first twelve chains in Table I were contributed by Gruebler [6], and the last four chains are said to have been contributed by Alt [7]. Klien [8] and Hain [9] examined these sixteen chains and pointed out that the first twelve chains of Table I could be derived from Watt's and Stevenson's six-link chains by adding to these chains a dyad consisting of two binary links and three revolute pairs. A systematic analysis by Hain and Zeilstroff [10, 11] shows that these sixteen eight-link chains with a single joint yield additional 44 eight-link chains with

*References cited are shown in Bibliography.

TABLE I

MULTI-LOOP EIGHT-LINK KINEMATIC CHAINS
WITH ONE DEGREE OF FREEDOM
(SEE CROSSLEY [5])



multiple joints. Using graph theory, Huang [12] performed structural synthesis of eight-link chains and enumerated, as shown in Table II, a total of 3,511 eight-link chains with kinematic pairs such as revolute pairs, and cam pairs.

Kinematic inversions from 60 chains with revolute pairs yield a total of 335 mechanisms with single and multiple joints. These mechanisms are further classified by Hain and Zeilstroff [13-16] into two groups. The first group consists of 445 cases in which the level of coupler planes** of eight-link mechanisms is higher than that of a four-link mechanism. The second group consists of 215 cases in which the output link of an eight-link mechanism is expected to have its angle of oscillation larger than that of a four-link mechanism.

Besides classifying, Hain also investigated coupler curves of selected eight-link mechanisms [9]. Analysis of these coupler curves led Hain [10] to synthesize eight-link mechanisms either to produce a larger angle of oscillation or to produce multiple instantaneous or finite dwells.

Geometric properties of coupler curves of eight-link mechanisms obtained by adding a dyad to Stevenson's and Watt's six-link mechanisms were investigated by Primrose, Freudenstein, and Roth [17]. Other notable work on properties of higher coupler curves of an eight-link mechanism was done by Wunderlich [18].

The subject of multigeneration of coupler curves of multi-loop mechanisms has become a rewarding one ever since Roberts [19] proposed the multigeneration theorem for a four-link mechanism. Notable

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**See Hain [13].

TABLE II

ENUMERATION OF EIGHT-LINK CHAINS WITH REVOLUTE, PRISM AND CAM PAIRS [12]

P = Prism Pairs
R = Revolute Pairs
C_a= Cam Pairs

Kinematic Chains With	No. of Chains
R (Single Joint)	16
R (Double Joints)	44
1P + 9R	88
2P + 8R	349
3P + 7R	810
4P + 6R	1,157
5P + 5R	730
6P + 4R	174
$1C_a$ + 8R	38
$2C_a$ + 6R	58
$3C_a$ + 4R	35
$4C_a$ + 2R	11
$5C_a$	1
Total	3,511

contributions relevant to the study of eight-link mechanisms include Roth [20] , Rischen [21], Gibson [22], Dijksman [23-25], and Soni [26-31]. Rischen [21] proposed multigeneration theorem for the Stevenson's six-link mechanism with three fixed pivots. Roth [20] extended Rischen's technique to propose the multigeneration theorem for eight-link and other similar multi-loop mechanisms. Soni [26-31] proposed an extension of stretch-rotation technique to discover coupler cognate mechanisms of six and eight-link mechanisms with parallelogram loops. The method proposed by Dijksman [22-24] for the general case of six-link mechanisms is extended by Soni [31] to obtain coupler cognate mechanisms of a class of eight-link mechanisms. The principle of inversion followed by the application of stretch-rotation has proved to be the key to the existence of eight-link cognate mechanisms [31-34].

The unusual application of cognate four-link mechanisms was first proposed by Hain [35] to synthesize Watt's six-link mechanisms for generation of parallel motion. The design of eight-link mechanisms for generation of parallel motion can be similarly achieved by adding two cognate six-link mechanisms. Hain [10] has enumerated 58 cases of eight-link mechanisms with single joints which can be synthesized to generate parallel motion. The results of some of the intuitively designed eight-link mechanisms for generation of parallel motion are also reported by Sylvester [36].

Synthesis of eight-link mechanisms to coordinate motions of input, output, or coupler links is known to be carried out using either graphical or analytical techniques.

Notable contributions in synthesis of eight-link mechanisms using graphical techniques include the works of Mueller [37-41], Kiper [42], Ihme [43], Wetzel [44, 45], Ludwig [46], and Hain [47]. The graphical techniques contributed by these kinematicians led to the design of eight-link mechanisms for a few precision positions of coupler, input, or output links.

The survey of the existing literature shows that, even though kinematicians in the past have shown considerable interest in the study of eight-link mechanisms, the design of all possible eight-link mechanisms for a variety of motion programs involving coordinations of input, output, or coupler links is not fully exploited.

In this thesis analytical approach has been adopted to synthesize eight-link mechanisms for the following design situations:

1. Coupler Point-Path Generation.

2. Coupler Point-Path Generation Coordinated with the Angular Displacements of Input Link.

3. Coupler Point-Path Generation Coordinated with the Angular Displacements of Input and Output Links.

4. Rigid Body Guidance.

5. Rigid Body Guidance coordinated with the Angular Displacements of the Input Link.

6. Rigid Body Guidance coordinated with the Angular Displacements of the Input and Output Links.

7. Coordination of Angular Displacements of Input and Output Links.

8. Generation of Two Coupler-Points Paths.

9. Non-rectilinear Motion Generation:

    Case 1. Synthesis of Hain's eight-link mechanism,

    Case 2. Synthesis of eight-link mechanism having five links in each of its loops.

10. Rectilinear Motion Generation.

Chapter II deals with the method of synthesis of planar mechanisms using displacement matrices. In this a general outline has been provided to obtain the design equations. The following chapter is concerned about the Marquardt's optimization technique which has been utilized to solve the system of design equations. These design equations are derived in Chapter IV for the first 8 problems discussed above. This chapter also includes numerical examples to illustrate the technique.

Unlike the first eight problems the ninth problem can be solved in closed form. This is done by using the principle of Linear Superposition. The application of this powerful principle has been discussed in Chapter V. Following chapter is the application of this theory, developed in Chapter V, to the problem of simultaneous guidance of two rigid bodies. Chapter VII summarizes and concludes the treatment.

The Appendix contains two programs. One for point-path generation and the other for two rigid bodies guidance. Programs for other problems may be developed by making some minor changes in these two programs.

TABLE III

THE EIGHT-LINK MECHANISMS AS DERIVED BY HAIN [11]

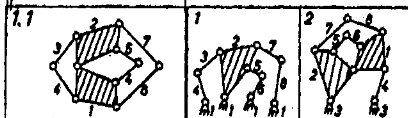
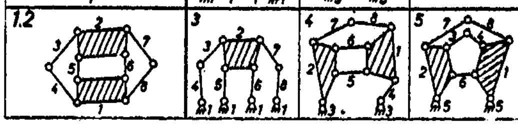
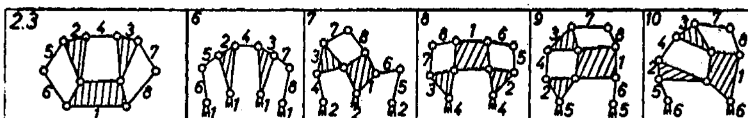
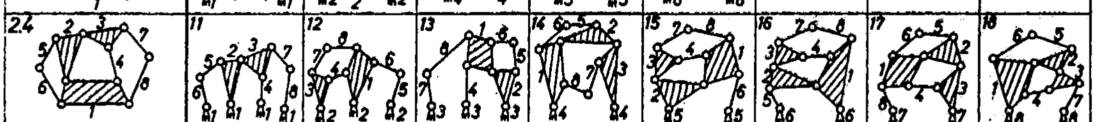
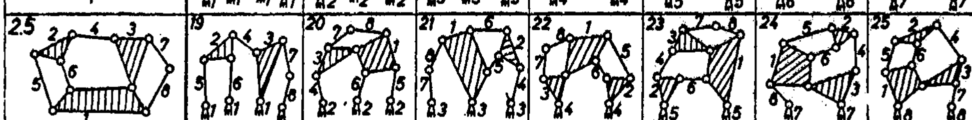
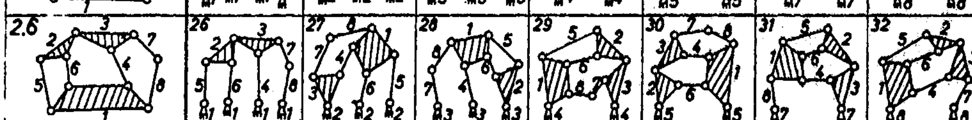
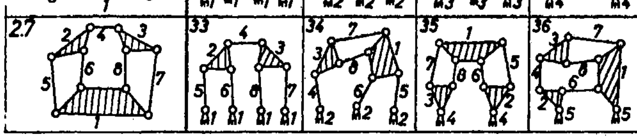
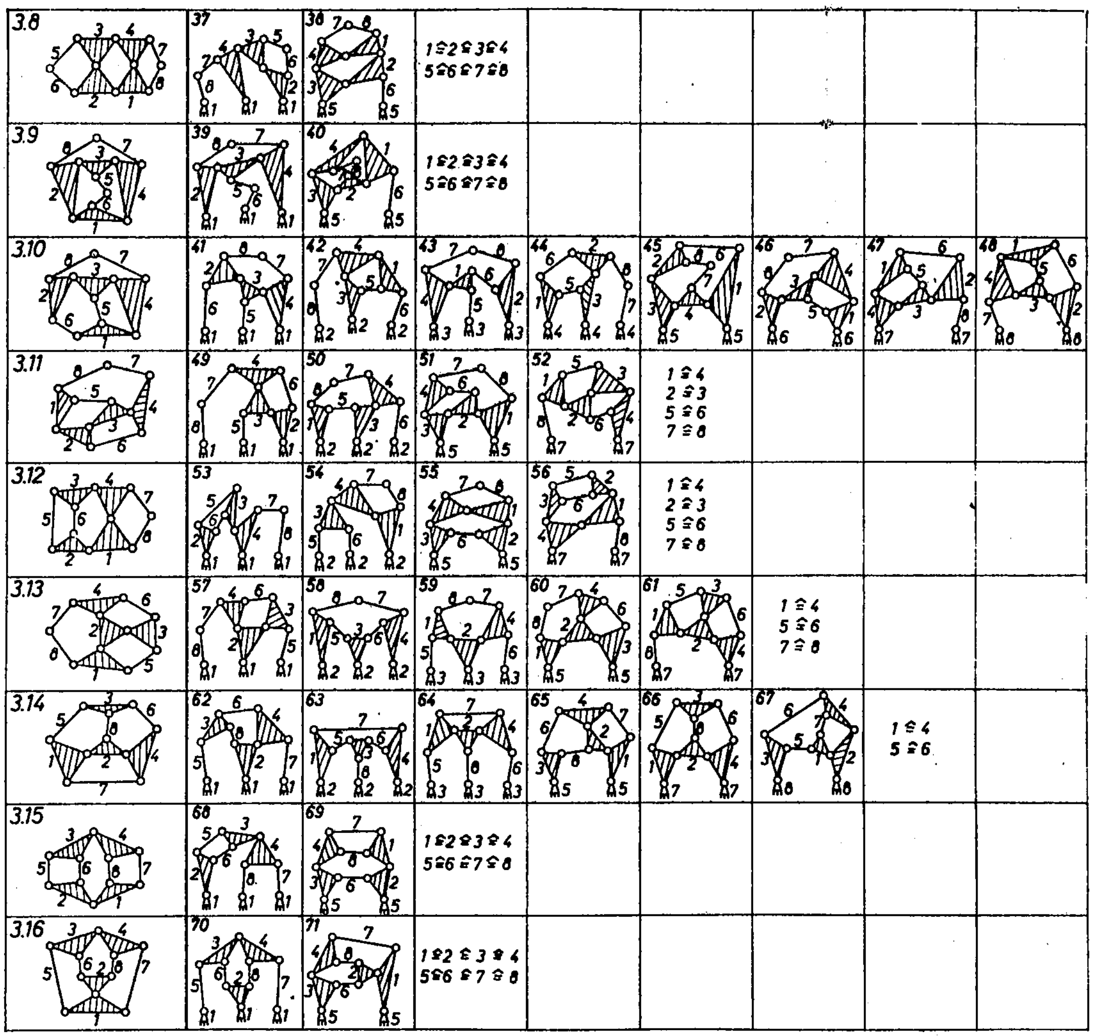
Eight-link Chains	Eight-link Mechanisms	
1.1		1≙2 3≙4≙5≙6≙7≙8
1.2		1≙2 5≙6 3≙4≙7≙8
2.3		2≙3 5≙7 6≙8
2.4		
2.5		5≙6
2.6		5≙6
2.7		2≙3 5≙6≙7≙8

TABLE III (Continued)



CHAPTER II

GENERAL METHOD OF DESIGN OF MECHANISMS USING DISPLACEMENT MATRICES

The synthesis of a mechanism begins with an engineering need. This need can be mathematically expressed as the functional specifications. The functional specifications are the necessary information a designer needs to carry out the design. The designer can then adopt one of the numerous graphical and analytical approaches available. Among the various analytical approaches the one using displacement matrices has been adopted here. The following is a detailed description of this matrix approach.

Consider a case wherein all the positions of points P and B; and the angular displacements of link PA, i.e. θ_{1n} are known. See Figure 1. It is required to find the coordinates of the pivot A in its first position A_1.

Let $\overline{P}_i \triangleq [X_{pi}\ Y_{pi}\ \ 1\ ]^T$ denote the position vector of a point $P(X_{pi}, Y_{pi}, \ 1\ )$ in a cartesian coordinate system. And let the displacement matrix for planar motion, as defined in [49], be represented by $[D(\overline{p}_1, \overline{p}_n, \theta_{1n})]$ where

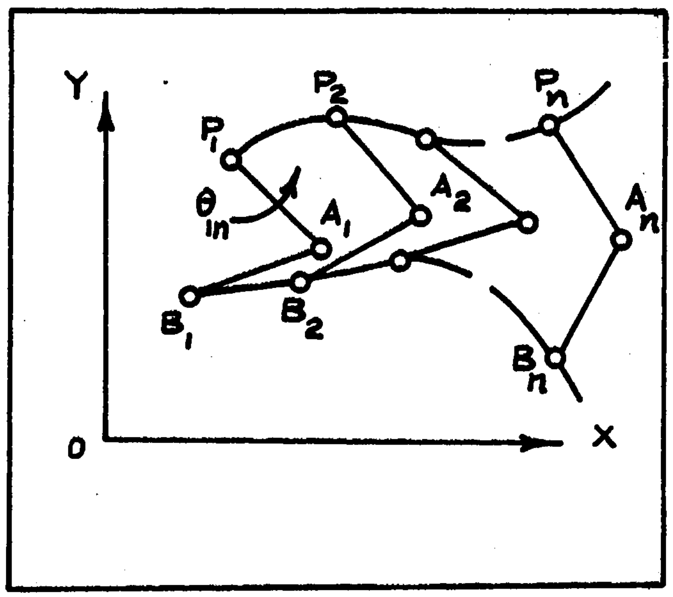


Figure 1. Synthesis Procedure

$$[D(\bar{p}_1, \bar{p}_n, \theta_{1n})] = \begin{bmatrix} \cos\theta_{1n} & -\sin\theta_{1n} & X_{pn} - X_{p1}\cos\theta_{1n} + Y_{p1}\sin\theta_{1n} \\ \sin\theta_{1n} & \cos\theta_{1n} & Y_{pn} - X_{p1}\sin\theta_{1n} - Y_{p1}\cos\theta_{1n} \\ 0 & 0 & 1 \end{bmatrix} \quad (2\text{-}1)$$

Now the n^{th} position of the pivot A may be expressed in terms of its first position as

$$\bar{a}_n = [D(\bar{p}_1, \bar{p}_n, \bar{\theta}_{1n})]\, \bar{a}_1 \quad (2\text{-}2)$$

where n = 2, 3, ..., m.

Using the geometry of constrained motion, we can write

$$(\text{length } A_nB_n)^2 - (\text{length } A_1B_1)^2 = 0$$

$$(\bar{a}_n - \bar{b}_n)^T (\bar{a}_n - \bar{b}_n) - (\bar{a}_1 - \bar{b}_1)^T (\bar{a}_1 - \bar{b}_1) = 0 \quad (2\text{-}3)$$

For m precision positions, Eq. (2-3) describes a system of (m-1) design equations which can be numerically solved for the unknown coordinates X_{a1}, Y_{a1} of point A_1.

In some cases the angles θ_{1n} or the positions of B may not be known. Then, θ_{1n} or coordinates of point B_1 are treated as unknown parameters in the above system of non-linear algebraic design equations.

CHAPTER III

MARQUARDT'S OPTIMIZATION TECHNIQUE

In general the system of design equations is highly non-linear. Hence, a closed form solution is not practical. The usual practice, then, is to solve the system of equations, for specific values of known parameters involved, numerically on a computer. There are many numerical techniques which find diverse application. Among the relatively new techniques is the Marquardt's optimization technique which has been used in the present study. And hence a brief description of this useful technique is quite relevant.

The problem of solving a system of nonlinear algebraic equations of the form

$$\underline{f}(\underline{z}) = \underline{0}$$

where $\underline{z}$ is the vector of n unknown parameters, is equivalent to finding $\underline{z}$ which minimizes a function Φ given by

$$\Phi = \underline{f}^T\underline{f}.$$

Marquardt's method [50] is found suitable for this class of optimization problem. This method attempts to improve the rate of convergence by interpolating between Newton-Ralphson method and the gradient method. This is accomplished by rotating the correction vector $\underline{\delta}$ through an appropriate angle σ away from $\underline{\delta}_g$, the negative gradient vector of Φ.

The algorithm to find the extremum comprises of the following steps for the γth iteration:

(1) Compute
$$[P^{(\gamma)}] = [\frac{\partial f^{(\gamma)}}{\partial \underline{z}^{(\gamma)}}]$$
$$[A^{(\gamma)}] = [P^{(\gamma)}]^T [P^{(\gamma)}]$$
$$\underline{g}^{(\gamma)} = -[P^{(\gamma)}]^T \underline{f}^{(\gamma)}.$$

(2) Scale down the matrix $A^{(\gamma)}$ and the vector $\underline{g}^{(\gamma)}$ as:
$$[A^{*(\gamma)}] = (a^*_{jk}) = \frac{a_{jk}}{\sqrt{a_{jj}}\,\sqrt{a_{kk}}}$$
$$\underline{g}^{*(\gamma)} = (g^*_j) = (\frac{g_j}{\sqrt{a_{jj}}})$$
where j, k = 1, 2, ..., n.

(3) Find the vector $\underline{\delta}^{*(\gamma)}$ from
$$[A^{*(\gamma)} + \lambda^{(\gamma)}\ I]\ \underline{\delta}^{*(\gamma)} = \underline{g}^{*(\gamma)}.$$
(The value of $\lambda^{(\gamma)}$ is generated in the later part of the algorithm. But initially $\lambda^{(0)}$ has to be specified some value. A value of 100 is found effective on the synthesis problem under consideration, though Marquardt suggests 0.01.)

(4) Compute the correction vector and then the new trial vector:
$$\delta_j^{(\gamma)} = \delta_j^{*(\gamma)}/\sqrt{a_{jj}} \qquad j = 1, 2, \ldots, n$$
$$\underline{z}^{(\gamma+1)} = \underline{z}^{(\gamma)} + \underline{\delta}^{(\gamma)}.$$
Except at the optimum there always exists a $\lambda^{(\gamma)}$ sufficiently

large so as to result in

$$\Phi^{(\gamma+1)} < \Phi^{(\gamma)}. \qquad (2\text{-}4)$$

Also it is possible to minimize Φ by considering it as a function of the variable λ. But a better global strategy, as suggested by reference [50], is to use as small a value of λ as is permissible while keeping inviolate the constraint (2-4). To accomplish this, we will adopt the following procedure:

Let $u > 1$ (say $u = 5$). Provide the angle criterion by specifying the upper bound on cos σ where σ is, as defined earlier, the deviation of the correction vector from the gradient direction given by the Equation (2-5).

(5) Compute:

$$\cos(\gamma) = \frac{\underline{\delta}^{*(\gamma)^T} \underline{g}^{*(\gamma)}}{\|\underline{\delta}^{*(\gamma)}\| \cdot \|\underline{g}^{*(\gamma)}\|} \qquad (2\text{-}5)$$

(6) a. Compute $\Phi(\lambda^{(\gamma-1)})$. If $\lambda^{(\gamma-1)} \ll 1$, then skip the steps b and c.

b. Compute $\Phi(\lambda^{(\gamma-1)}/u)$.

c. If $\Phi(\lambda^{(\gamma-1)}/u) \leq \Phi(\lambda^{(\gamma)})$ then let $\lambda^{(\gamma)} = \lambda^{(\gamma-1)}/u$.

d. If $\Phi(\lambda^{(\gamma-1)}/u) > \Phi(\lambda^{(\gamma)})$, and $\Phi(\lambda^{(\gamma-1)}) \leqq \Phi(\lambda^{(\gamma)})$ then let $\lambda^{(\gamma)} = \lambda^{(\gamma-1)}$.

e. If $\Phi(\lambda^{(\gamma-1)}/u) > \Phi(\lambda^{(\gamma)})$ and $\Phi(\lambda^{(\gamma-1)}) > \Phi(\lambda^{(\gamma)})$ increase λ by successive multiplication until:

(i) For some smallest w, $\Phi(\lambda^{(\gamma-1)}u^w) \leqq \Phi(\lambda^{(\gamma)})$
Let $\lambda^{(\gamma)} = \lambda^{(\gamma-1)} u^w$, or

(ii) If $\cos\sigma^{(\gamma)}$ exceeds the limit provided let $\underline{z}^{(\gamma+1)} = \underline{z}^{(\gamma)} + k^{(\gamma)}\,\delta^{(\gamma)}$, and choose $k^{(\gamma)}$ (by trial and error) sufficiently small so that $\Phi^{(\gamma+1)} < \Phi^{(\gamma)}$.

One is required to repeat the above steps until the value of the function Φ becomes sufficiently small. In addition to this condition, one may use other convergence criterion. Depending upon the admissibility of the design data, there may or may not be a $\Phi_{min} = 0$. In such a situation, the design specifications may have to be altered. In the numerical examples given in this thesis the minimum Φ ranges from 0.001 to 0.0001.

CHAPTER IV

DESIGN OF EIGHT LINK MECHANISM

The first eight problems described in Chapter I are considered here. The 9th problem is treated in Chapter VI.

Description of the Eight-Link Mechanism

Figure 10 shows the eight-link mechanism which is to be synthesized for various design situations. M and Q are the fixed pivots and A, B, C, D, E, F, G, and H are the moving pivots. P and S are the coupler points on links FE and HG. MAB is considered as the input link and QDG is the output link. Let α_{1n}, β_{1n}, γ_{1n}, δ_{1n}, θ_{1n}, η_{1n}, and ϕ_{1n} represent the angular displacements of the links FEP, CDE, AFH, BC, MAB, HGS and QDG measured from their first position to the n^{th} position as shown in Figure 10.

Design Problem 1: Coupler Point-Path Generation

Figure 3 shows a curve Z which can be approximately described using a set of k finitely separated positions P_1, P_2, ..., P_k. It is desired to pass the coupler point of the eight link mechanism through the points P_1, P_2, ..., P_k by appropriately selecting the mechanism parameters.

For the eight-link mechanism shown in Figure 10, the coupler point P is required to pass through a number of precision positions

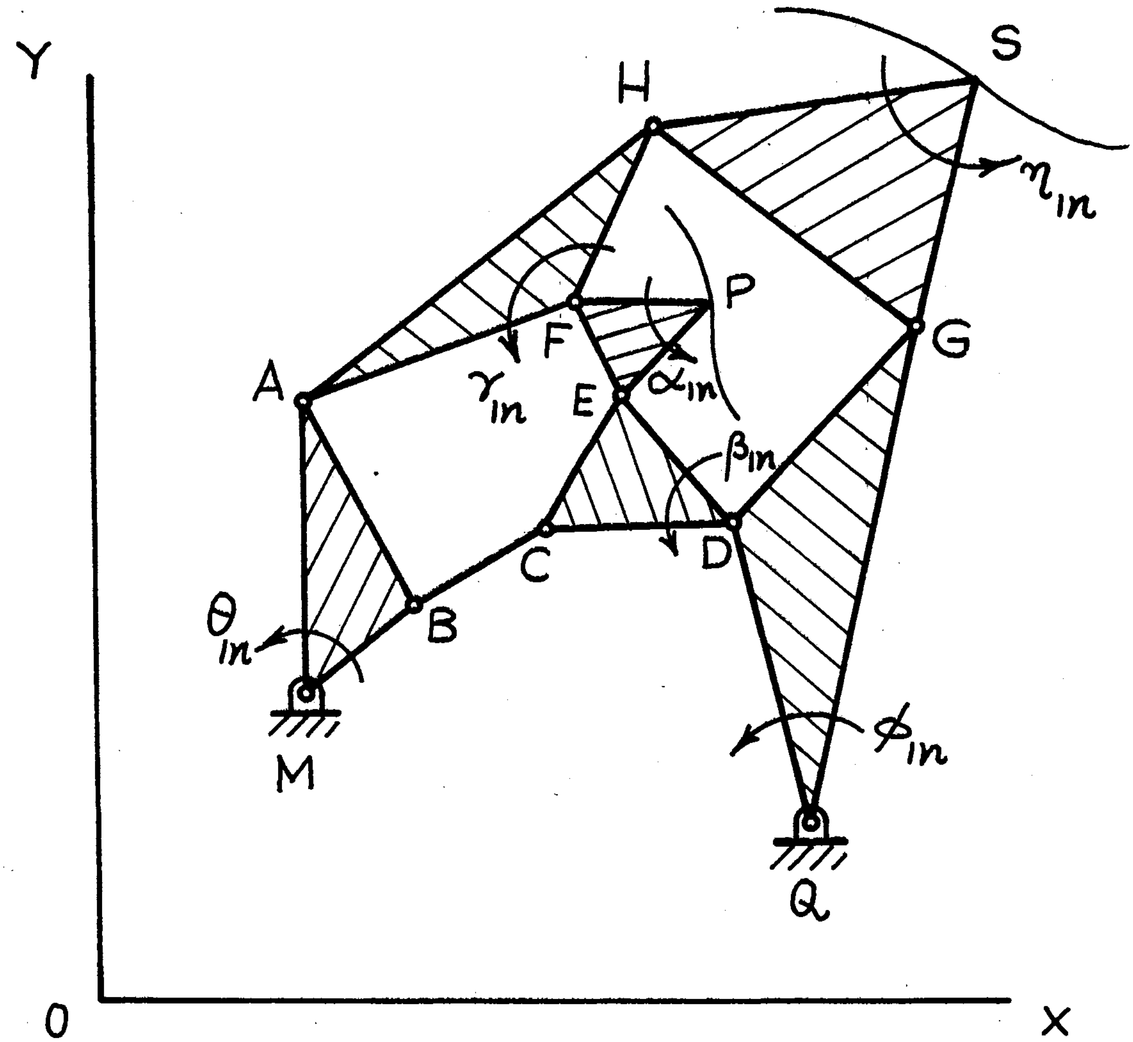


Figure 2. Eight-Link Mechanism with Five Links in All of Its Loops

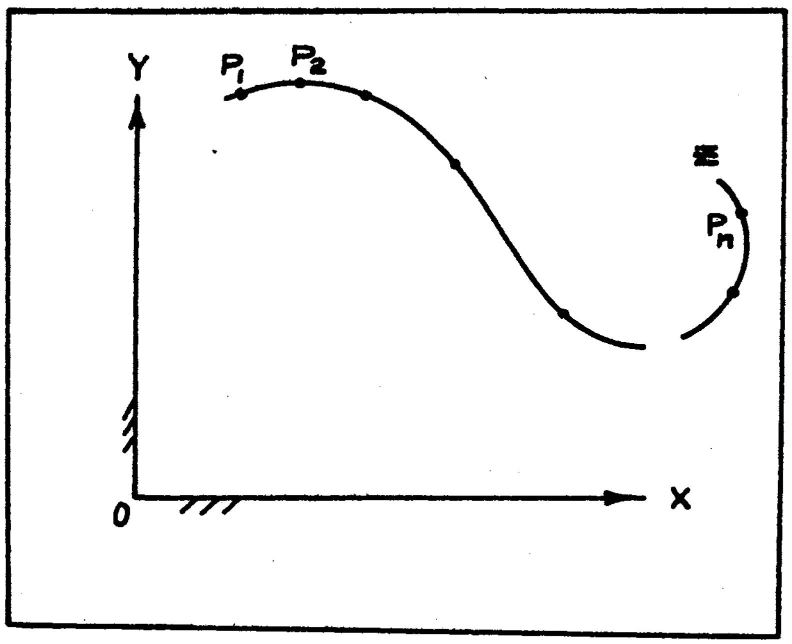


Figure 3. Point-Path Generation

which are specified. Let $\theta_{1n}, \gamma_{1n}, \Phi_{1n}$ and β_{1n} be treated as unknown parameters. The following steps yield the design equations:

(1) Express the n^{th} positions of the pivots A and B in terms of their first positions as,

$$(\bar{a}_n \; \bar{b}_n) = [D(\bar{m}, \bar{m}, \theta_{1n})] \; (\bar{a}_1 \; \bar{b}_1) \tag{4-1}$$

where n = 2, 3, etc. $[D(\bar{m}, \bar{m}, \theta_{1n})]$ is obtained from Equation (2-1).

(2) Express the n^{th} positions of the pivots F and H in terms of their first positions as

$$(\bar{f}_n \; \bar{h}_n) = [D(\bar{a}_1, \bar{a}_n, \gamma_{1n})] \; (\bar{f}_1 \; \bar{h}_1) \tag{4-2}$$

where n = 2, 3, etc. $[D(\bar{a}_1, \bar{a}_n, \gamma_{1n})]$ is obtained from Equation (2-1).

(3) Because of the symmetry of the eight-link mechanism about the coupler FE, we have

$$(\bar{d}_n \; \bar{g}_n) = [D(\bar{q}, \bar{q}, \Phi_{1n})] \; (\bar{d}_1 \; \bar{g}_1) \tag{4-3}$$

$$(\bar{c}_n \; \bar{e}_n) = [D(\bar{d}_1, \bar{d}_n, \beta_{1n})] \; (\bar{c}_1 \; \bar{e}_1) \tag{4-4}$$

(4) The kinematic constraints imposed by the links BC, FE, FP, EP and HG are given by the constant length condition as

$$(\bar{u}_n - \bar{v}_n)^T (\bar{u}_n - \bar{v}_n) - (\bar{u}_1 - \bar{v}_1)^T (\bar{u}_1 - \bar{v}_1) = 0 \tag{4-5}$$

where n = 2, 3, ...; and $\bar{u}$, $\bar{v}$ take the values as

$(\bar{u}, \bar{v}) = (\bar{b}, \bar{c}), (\bar{f}, \bar{e}), (\bar{f}, \bar{p}), (\bar{e}, \bar{p}), (\bar{h}, \bar{g})$.

Equation (4-5) represents a system of 5(n - 1) design equations involving 20 + 4(n - 1) unknown parameters which include

20 coordinates of the ten pivots and 4(n - 1) angles θ_{1n}, Φ_{1n}, γ_{1n}, β_{1n}. Therefore, the number of variables to be specified is 21 - n. Hence, a maximum of 21 precision positions of a coupler point may be specified without specifying any of the unknown parameters. For this case (i.e., n = 21) we have 100 design equations in 100 unknown parameters. Line 1 of Table IV presents a brief summary of this type of synthesis problem. Using the numerical technique, as presented in the Chapter II, the 100 design equations are solved for the illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table V. Table V shows the design specifications and the designed mechanism. The computer program for this problem is given in Appendix.

Design Problem 2: Coupler Point-Path Generation Coordinated with the Angular Displacements of Input Link

Figure 4 shows positions P_1, P_2, ..., P_n of a point P tracing path Z and the input link MA in its n positions MA_1, MA_2, ..., MA_n. In this case the eight-link mechanism is required to coordinate the motion of the input link in addition to that of the coupler point.

Figure 2 shows the eight-link mechanism with coupler point P and input link MA. The positions of the coupler point P (i.e., $\overline{p}_n$) and the rotations of the input link (θ_{1n}) are prescribed. The procedure for obtaining the design equations is exactly the same as for the previous problem. The only difference in the situation is that in Equations (4-5), θ_{1n} is now a known variable. Eq. (4-5)

TABLE IV

SUMMARY OF SYNTHESIS PROBLEMS

S. No.	Problem Specifications	Angles Given	Angles To Be Determined	Design Equations Obtained from the Constraint on the Links	No. of Equations	No. of Unknown Parameters	Max. n	Remarks
1	Path Generation	--	$\theta_{1n}, \gamma_{1n}, \Phi_{1n}, \beta_{1n}$	BC, FE, FP, EP, HG	5(n-1)	20 + 4(n-1)	21	Coupler point S is to be ignored
2	Path generation with input coordination	θ_{1n}	$\gamma_{1n}, \Phi_{1n}, \beta_{1n}$	BC, FE, FP, EP, HG	5(n-1)	20 + 3(n-1)	11	Ignore S
3	Path generation with input output coordination	θ_{1n}, Φ_{1n}	γ_{1n}, β_{1n}	BC, FE, FP, EP, HG	5(n-1)	20 + 2(n-1)	7	Ignore S; Two of the parameters are to be specified
4	Rigid-body guidance	α_{1n}	$\theta_{1n}, \delta_{1n}, \Phi_{1n}, \eta_{1n}$	AF, FH, HA, CD, DE, EC	6(n-1)	20 + 4(n-1)	11	Ignore S
5	Rigid-body guidance coordinated with input rotation	θ_{1n}, α_{1n}	$\delta_{1n}, \Phi_{1n}, \eta_{1n}$	AF, FH, HA, CD, DE, EC	6(n-1)	20 + 3(n-1)	7	Ignore S; Two of the parameters are to be specified

TABLE IV (continued)

S. No.	Problem Specifications	Angles Given	Angles To Be Determined	Design Equations Obtained from the Constraint on the Links	No. of Equations	No. of Unknown Parameters	Max. n	Remarks
6	Rigid-body guidance co-ordinated with the rotations of input and output	θ_{1n}, Φ_{1n}, α_{1n}	δ_{1n}, η_{1n}	AF, FH, HA, CD, DE, EC	6(n-1)	20 + 2(n-1)	6	Ignore S
7	Input-Output Coordination	θ_{1n}, Φ_{1n}	γ_{1n}, β_{1n}	BC, FE, HG	3(n-1)	16 + 2(n-1)	17	Ignore P and S. M (0,0), Q (1,0)
8	Guidance of two coupler points (on different links)	—	θ_{1n}, γ_{1n}, Φ_{1n}, β_{1n}	BC, FE, FP, EP, HG, HS, GS	7(n-1)	20 + 4(n-1)	7	Two variables are to be specified

TABLE V

DESIGNED MECHANISM FOR PATH GENERATION

Design Specifications:

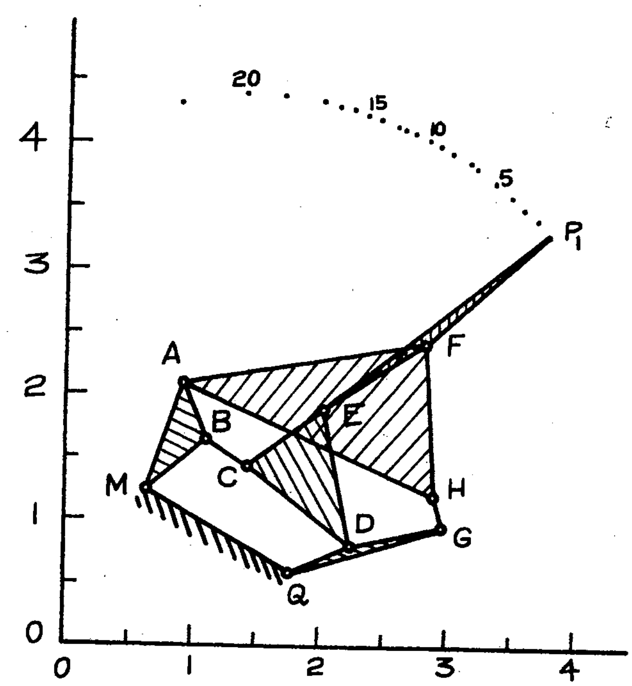
n	1	2	3	4	5	6	7	8	9	10
X_{pn}	3.77	3.68	3.57	3.47	3.34	3.19	3.14	3.00	2.90	2.82
Y_{pn}	3.27	3.40	3.49	3.58	3.69	3.80	3.84	3.92	3.97	4.02

11	12	13	14	15	16	17	18	19	20	21
2.70	2.62	2.56	2.43	2.33	2.21	2.10	1.88	1.66	1.37	0.86
4.07	4.11	4.14	4.19	4.22	4.26	4.29	4.33	4.36	4.37	4.32

Designed Mechanism:

	M	A	B	C	D
X-Coord.	0.62915	0.91971	1.10151	1.41327	2.25314
Y-Coord.	1.24016	2.70208	1.62616	1.42877	0.78881

	E	F	G	H	Q
X-Coord.	2.05385	2.82348	2.97356	2.90137	1.76447
Y-Coord.	1.86544	2.40824	0.90596	1.20064	0.57538



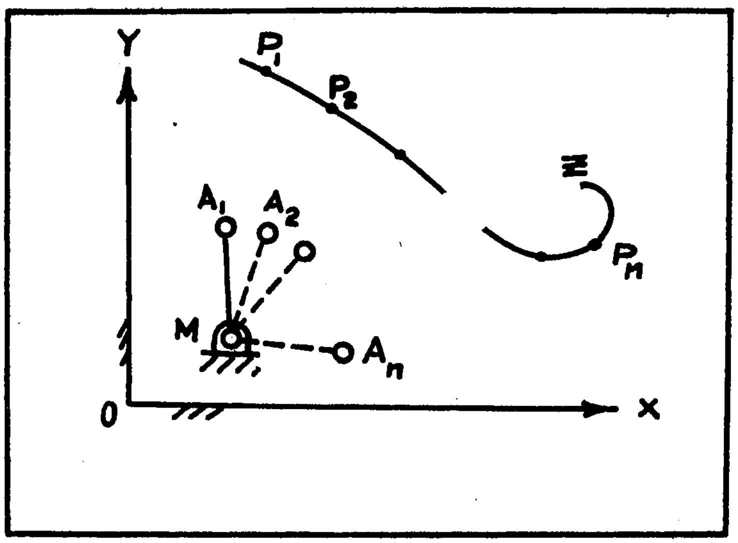


Figure 4. Point-Path Generation Coordinated with Angular Displacement of Input Link

describe a system of $5(n - 1)$ equations involving $20 + 3(n - 1)$ unknown parameters which include 20 coordinates of pivots when P is at P_1 and $3(n - 1)$ angles $\Phi_{1n}, \gamma_{1n}, \beta_{1n}$. While solving the system of equations for unknown parameters, we are required to specify $2(11 - n)$ variables. This shows that the maximum of 11 precision conditions may be specified without specifying any unknown parameters.

An illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table VI. Table VI shows the design requirements which the 11 positions of the coupler point and the 10 angular displacements. The designed mechanism is given by the coordinates of the various pivots. It also shows the sketch of the designed mechanism.

Design Problem 3: Coupler Point-Path Generation
Coordinated with Angular Displacements of
Input and Output Links

Figure 5 graphically displays this design situation. Here the coupler point is required to pass through points P_1, P_2, ..., P_n; and the input link MA and output link QD are required to execute the desired angular motions α_{1n} and β_{1n}.

Here the positions P_n of the coupler point P and θ_{1n}, and ϕ_{1n}, the angular displacements of the input and the output links are prescribed as design data (See Fig. 2). The system of $5(n - 1)$ equations given by Equation (4-5) is still valid. However, θ_{1n}, Φ_{1n} are known variables. Hence, the number of unknown parameters is reduced to $20 + 2(n - 1)$, (20 coordinates of pivots when P is at P_1

TABLE VI

DESIGNED MECHANISM FOR PATH GENERATION
WITH COORDINATED INPUT

Design Specifications:

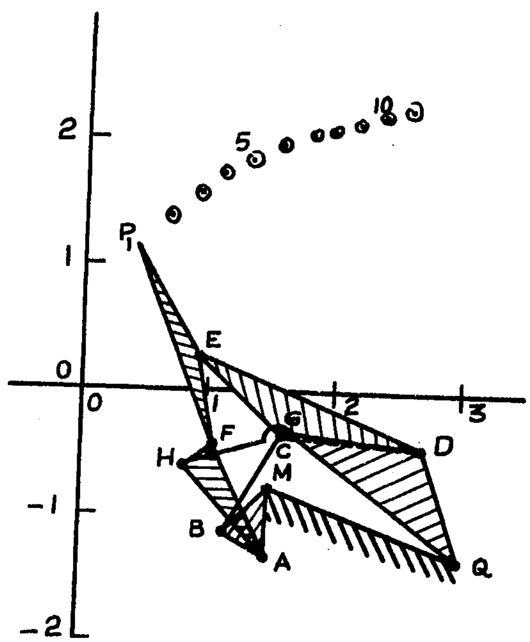
n	1	2	3	4	5	6
X_{pn}	0.43	0.67	0.90	1.08	1.30	1.54
Y_{pn}	1.11	1.39	1.57	1.73	1.83	1.95
θ_{1n}	0.0°	-33.0°	-56.0°	-74.0°	-89.0°	-107.7°

n	7	8	9	10	11
X_{pn}	1.76	1.94	2.13	2.33	2.54
Y_{pn}	2.03	2.08	2.13	2.19	2.25
θ_{1n}	-122.5°	-133.0°	-144.5°	-154.5°	-169.5°

Designed Mechanism:

	M	A	B	C	D
X-Coord.	[illegible]	[illegible]15	1.14882	1.60723	2.71015
Y-C[illegible]	-0.7[illegible]70	-1.32889	-1.14630	-0.35973	-0.44330

	E	F	G	H	Q
X-Coord.	0.92247	1.05881	1.65498	0.82220	3.01476
Y-Coord.	0.27922	-0.43551	-0.33240	-0.60595	-1.34682



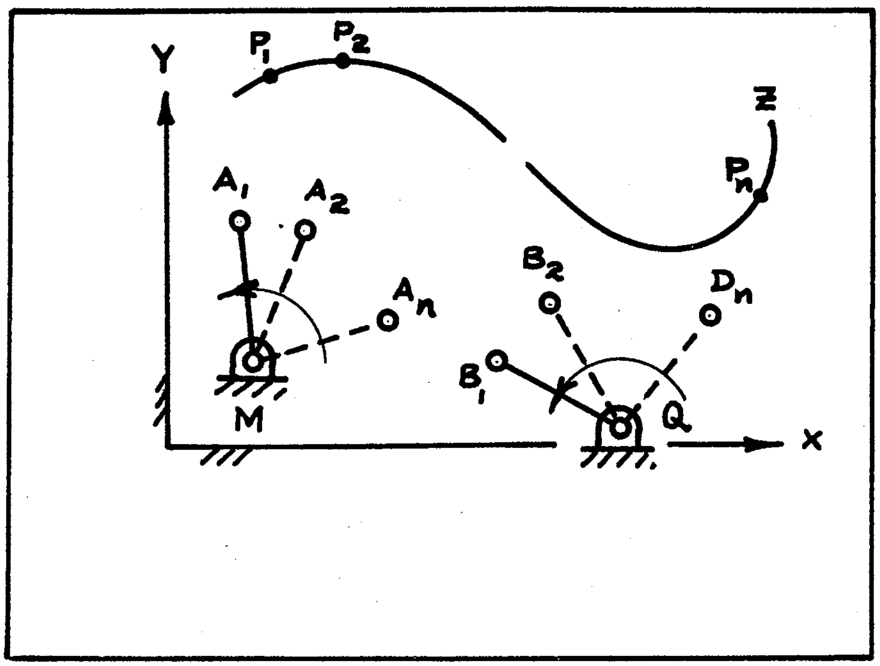


Figure 5. Point-Path Generation Coordinated with Angular Displacements of Input and Output Links

and 2 (n - 1) angles γ_{1n}, β_{1n}). So the number of variables to be specified is 23 - 3n. Hence a maximum of 7 precision conditions may be specified with any two of the unknown parameters specified.

An illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table VII, which shows the functional specifications and the numerically synthesized mechanism. The X and Y coordinates of pivot Q were assumed arbitrarily.

Design Problem 4: Rigid Body Guidance

In this case the coupler link is required to pass through a finite number of positions as shown in Figure 6. These positions are termed as "Precision Positions."

The body to be guided is located on the coupler link FE of the eight-link mechanism shown in Fig. 2. The various positions of the body, to be guided, are given by specifying the positions of a point P on the body and the angular displacements of the body. That is $\overline{p}_n$ and α_{1n} are specified as design requirements.

Let θ_{1n}, Φ_{1n}, δ_{1n} and η_{1n} be treated as unknown parameters. Then the following steps yield the design equations:

(1) Same as step (1) of problem 1 which results in Equation (4-1), i.e.,

$$(\overline{a}_n\ \overline{b}_n) = [D(\overline{m},\ \overline{m},\ \theta_{1n})]\ (\overline{a}_1\ \overline{b}_1) \qquad (4\text{-}1)$$

where n = 2, 3, ...

(2) Express the n^{th} position of the pivot C in terms of its first position as,

TABLE VII

DESIGNED MECHANISM FOR PATH GENERATION
WITH COORDINATED INPUT
AND OUTPUT

Design Specifications:

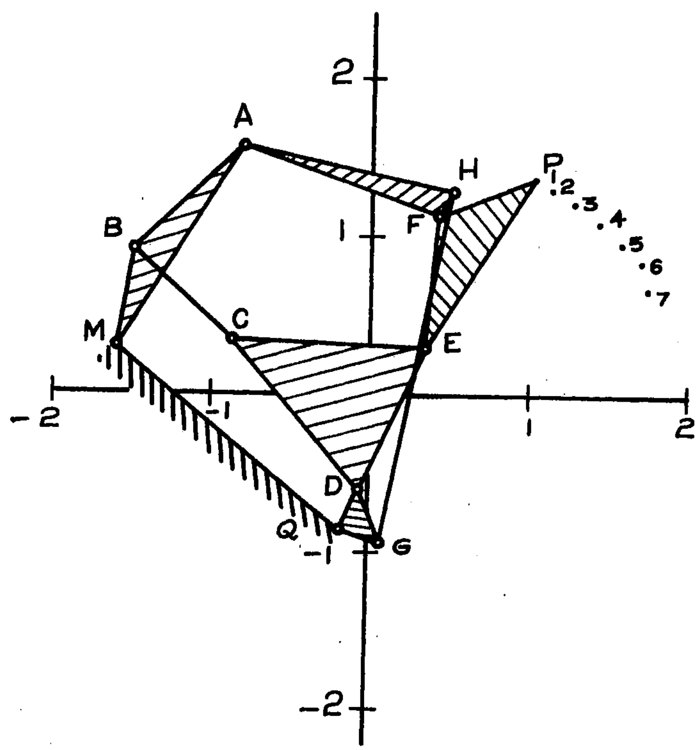
n	1	2	3	4	5	6	7
X_{pn}	1.01	1.13	1.27	1.43	1.57	1.70	1.73
Y_{pn}	1.37	1.30	1.21	1.09	0.96	0.85	0.67
θ_{1n}	0°	-4°	-10°	-18°	-29°	-43°	-62°
Φ_{1n}	0°	-4°	-8°	-13°	-18°	-28°	-73°

$X_q = -0.18$, $Y_q = -0.86$

Designed Mechanism:

	M	A	B	C	D
X-Coord.	-1.59859	-0.82915	-1.50046	-0.87986	-0.07811
Y-Coord.	0.38097	1.57140	0.91077	0.34802	-0.60932

	E	F	G	H	Q
X-Coord.	0.34770	0.42630	0.07960	0.50369	-0.18000
Y-Coord.	0.30579	1.14382	-0.94352	1.29994	-0.86000



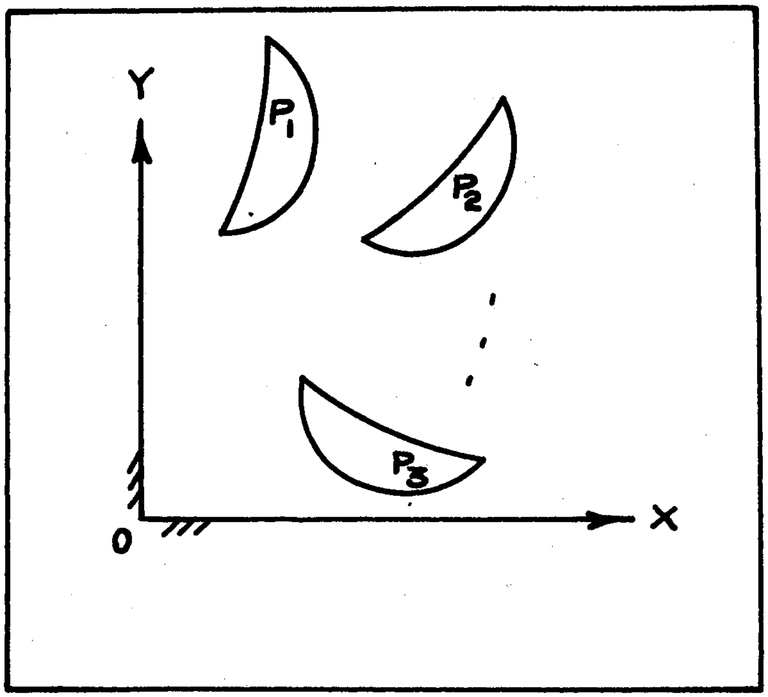


Figure 6. Rigid Body Guidance

$$\bar{c}_n = [D(\bar{b}_1, \bar{b}_n, \delta_{1n})]\, \bar{c}_1 \tag{4-6}$$

where n = 2, 3, ...

(3) Similarly on the other side, due to symmetry of the mechanism about the link FE, we have

$$(\bar{d}_n\ \bar{g}_n) = [D(\bar{q}, \bar{q}, \Phi_{1n})]\, (\bar{d}_1\ \bar{g}_1) \tag{4-3}$$

and

$$\bar{h}_n = [D(\bar{g}_1, \bar{g}_n, \eta_{1n})]\, \bar{h}_1 \tag{4-7}$$

where n = 2, 3, ...

(4) The kinematic constraints imposed by the links AF, AH, FH, CD, CE and DE are given by the constant length condition as

$$(\bar{u}_n - \bar{v}_n)^T (\bar{u}_n - \bar{v}_n) - (\bar{u}_1 - \bar{v}_1)^T (\bar{u}_1 - \bar{v}_1) = 0 \tag{4-8}$$

where n = 2, 3, ...; and $\bar{u}$, $\bar{v}$ take the values as

$$(\bar{u}, \bar{v}) = (\bar{a}, \bar{f}), (\bar{a}, \bar{h}), (\bar{f}, \bar{h}), (\bar{c}, \bar{d}), (\bar{c}, \bar{e}), (\bar{d}, \bar{e}).$$

Equations (4-8) when substituted from Equations (4-1), (4-6), (4-3) and (4-7) represent a system of 6(n - 1) design equations involving 20 + 4(n - 1) unknown design parameters which include 20 coordinates of the ten pivots when P is at P_1; and 4(n - 1) angles θ_{1n}, Φ_{1n}, δ_{1n}, and η_{1n}. Therefore, the number of variables to be specified is 2(11 - n). Hence a maximum of 11 precision positions of the body may be specified without specifying any unknown parameters.

An illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table VIII. The 11

TABLE VIII

DESIGNED MECHANISM FOR RIGID BODY GUIDANCE

Design Specifications:

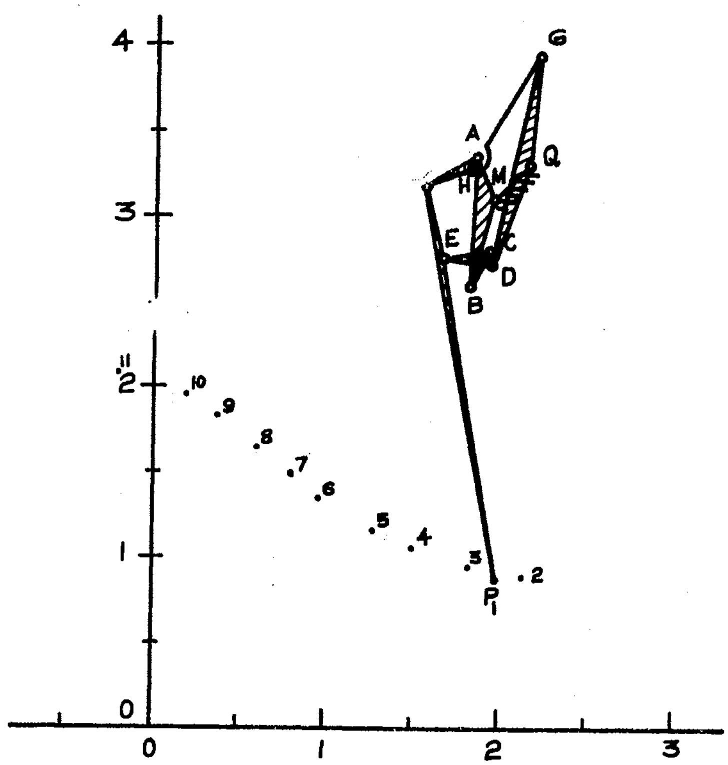
n	1	2	3	4	5	6
X_{pn}	1.990	2.135	1.825	1.500	1.270	0.96
Y_{pn}	0.880	0.890	0.950	1.050	1.165	1.350
α_{1n}	0.0^o	-11.0^o	-19.0^o	-28.0^o	-35.0^o	-45.0^o

n	7	8	9	10	11
X_{pn}	0.800	0.600	0.370	0.190	-0.200
Y_{pn}	1.485	1.650	1.830	1.950	2.070
α_{1n}	-50.0^o	-58.0^o	-64.0^o	-69.5^o	-75.5^o

Designed Mechanism:

	M	A	B	C	D
X-Coord.	1.96719	1.85242	1.82024	1.93878	1.95000
Y-Coord.	3.09646	3.33514	2.59954	2.78937	2.71429

	E	F	G	H	Q
X-Coord.	1.67836	1.57843	2.21511	1.82461	2.16106
Y-Coord.	2.75182	3.16626	3.92578	3.43749	3.29276



positions of the rigid body are specified by the coordinates of coupler point P and the angular displacements α_{1n}. The sketch of the designed mechanism shows the reasonably good link proportions.

Design Problem 5: Rigid Body Guidance Coordinated with Angular Displacements of Input Link

Figure 7 shows the specifications of this problem. As the rigid body moves through the specified positions P_1, P_2, etc., the input link MA rotates through positions MA_1, MA_2, etc. describing the specified angles.

In this problem the positions of the coupler point P, the rotations of the coupler link FE and the input link MAB are specified. Hence $\overline{P}_n$, θ_{1n}, α_{1n} are known quantities. Equations (4-8), derived in the preceding section, are still valid. But the only change will be that θ_{1n} is now a known parameter. Therefore the number of unknown parameters is now reduced to $23 + 3(n - 1)$. Then the number of variables to be specified is $21 - 3n$. So a maximum of 7 precision conditions may be specified with any two of the unknown parameters specified arbitrarily.

An illustrative design problem to synthesize an eight-link mechanism is presented in Table IX. The synthesis requirements and the synthesized mechanism are shown. The X and Y coordinates of Q were arbitrarily assumed.

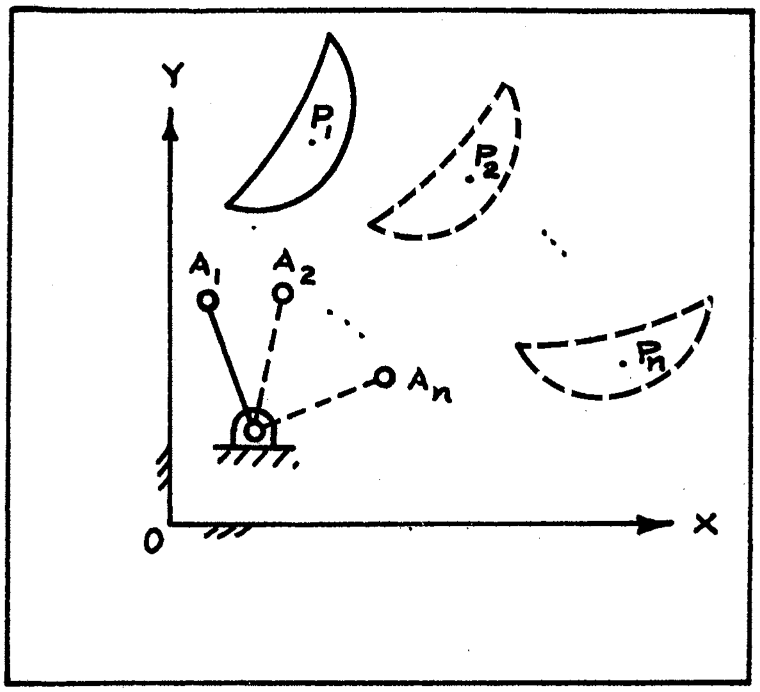


Figure 7. Rigid Body Guidance Coordinated with Angular Displacement of Input Link

TABLE IX

DESIGNED MECHANISM FOR RIGID BODY GUIDANCE
COORDINATED WITH INPUT
ROTATION

Design Specifications:

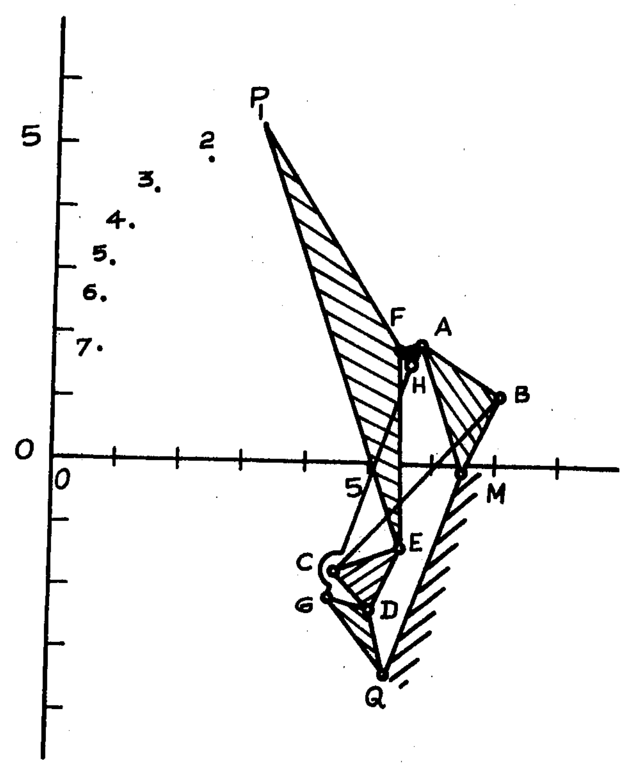
n	1	2	3	4	5	6	7
X_{pn}	3.25	2.39	1.55	1.15	0.87	0.72	0.71
Y_{pn}	5.32	4.75	4.22	3.68	3.08	2.50	1.72
α_{1n}	0.0	6.5°	13.0°	16.0°	19.0°	20.0°	24.0°
θ_{1n}	0°	16°	32°	44°	57°	70°	89°

$X_q = 5.32$, $Y_q = -3.40$

Designed Mechanism:

	M	A	B	C	D
X-Coord.	6.48702	5.80701	7.06589	4.49555	5.07659
Y-Coord.	-0.14000	1.86011	1.06823	-1.78726	-2.38622

	E	F	G	H	Q
X-Coord.	5.51154	5.47689	4.40349	5.64713	5.32000
Y-Coord.	-1.37559	1.78129	-2.18241	1.53520	-3.40000



Design Problem 6: Rigid Body Guidance Coordinated with Angular Displacements of Input and Output Links

In this case the various positions of rigid body located on a coupler link are specified along with the angular displacements of the input and output links. Hence in Figure 2 the positions of the coupler point P and the angular displacements θ_{1n}, Φ_{1n} and α_{1n} are specified. Equations (4-8) are still valid. But now the number of unknown parameters to be specified are $4(6 - n)$. Hence a maximum of 6 precision conditions may be specified without specifying any of the unknown parameters.

An illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table X, which shows the design requirements and the numerically generated solution.

Design Problem 7: Coordination of Angular Displacements of Input and Output Links

The angular displacements of the input link MA and output link QD are specified as indicated in Figure 8. The fixed link's length MQ may be arbitrarily assumed as unity because the functional characteristics, of the mechanism for this particular specification, are scale-invariant. That is if the mechanism is enlarged or reduced in scale then the input-output relationship of the mechanism remains unchanged.

In this case the angles θ_{1n} and Φ_{1n} are specified (See Figure 2). Notice that there are no coupler points P or S.

TABLE X

DESIGNED MECHANISM FOR RIGID BODY GUIDANCE
COORDINATED WITH THE ROTATIONS OF
INPUT AND OUTPUT LINKS

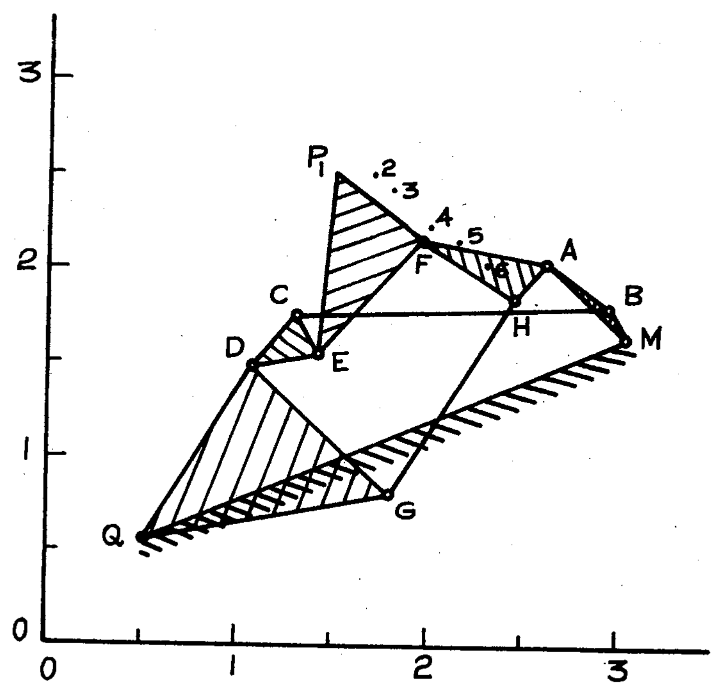
Design Specifications:

n	1	2	3	4	5	6
X_{pn}	1.50	1.70	1.80	2.00	2.15	2.30
Y_{pn}	2.50	2.50	2.35	2.25	2.15	2.02
α_{1n}	0^o	-10^o	-15^o	-30^o	-35^o	-50^o
θ_{1n}	0^o	-57^o	73^o	89^o	111^o	120^o
Φ_{1n}	0^o	-20^o	-25^o	-29^o	-30^o	-27^o

Designed Mechanism:

	M	A	B	C	D
X-Coord.	3.04151	2.62667	2.94529	1.30683	1.07885
Y-Coord.	1.64865	2.03078	1.80209	1.75794	1.49075

	E	F	G	H	Q
X-Coord.	1.42132	1.96418	1.80506	2.46686	0.51126
Y-Coord.	1.55678	2.15470	0.81551	1.86444	0.56664



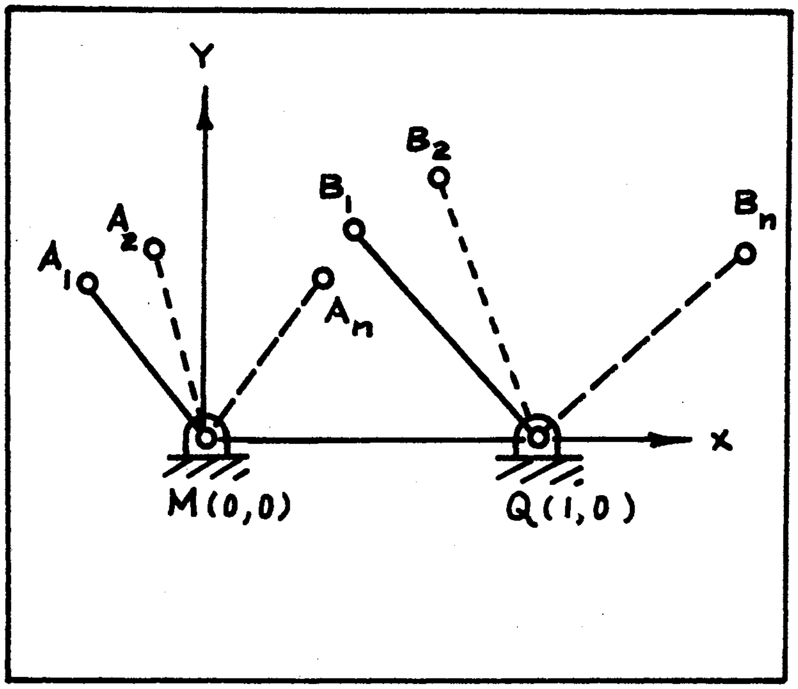


Figure 8. Coordination of Angular Displacements of Input and Output Links

The steps (1), (2), and (3) of problem 1 are common to this problem. The step (4) in this case is, obtaining the design equations from the constant length conditions of links BC, FE and HG as

$$(\bar{u}_n - \bar{v}_n)^T (\bar{u}_n - \bar{v}_n) - (\bar{u}_1 - \bar{v}_1)^T (\bar{u}_1 - \bar{v}_1) = 0 \qquad (4\text{-}9)$$

where n = 2, 3, etc.; and $\bar{u}$, $\bar{v}$ take the values as

$$(\bar{u}, \bar{v}) = (\bar{b}, \bar{c}), (\bar{f}, \bar{e}), (\bar{h}, \bar{g}).$$

Equations (4-9) represent a system of 3(n - 1) design equations containing 16 + 2(n - 1) (16 coordinates of the 8 moving pivots; 2(n - 1) angles γ_{1n}, β_{1n}) unknown parameters. Note that M is assumed as (0,0) and Q as (1,0) points without any loss of generality.

Now the number of variables to be specified is 17 - n. Hence a maximum of 17 precision conditions may be specified without specifying any of the unknown parameters.

An illustrative design problem to synthesize an eight-link mechanism of Figure 2 is presented in Table XI. The 17 positions of the input and output link are arbitrarily chosen as design requirements. The designed mechanism is sketched to illustrate the link proportions.

Design Problem 8: Generation of Two
Coupler-Point Paths

Here two discretized point-paths are required to be traced by coupler points of two coupler links of eight-link mechanism. In Figure 2 points P and S are two such coupler points. So, the various positions of the coupler points P and S are known.

TABLE XI

DESIGNED MECHANISM FOR INPUT
OUTPUT COORDINATION

Design Specifications:

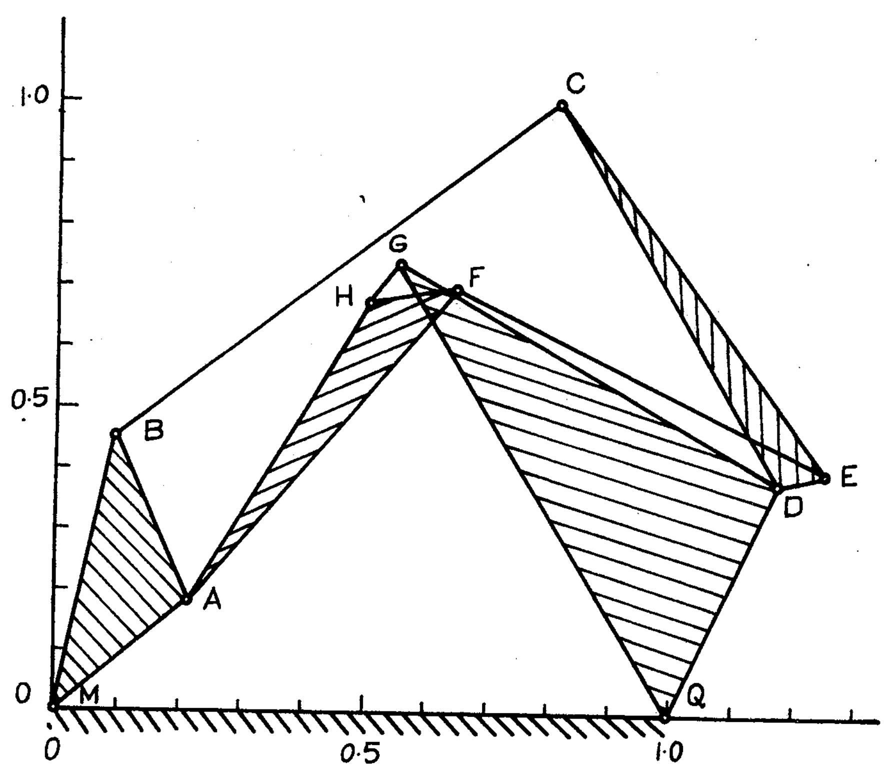
n	1	2	3	4	5	6	7	8	9
θ_{1n}	0.00°	30°	40°	50°	60°	70°	75°	80°	85°
Φ_{1n}	0.00°	10°	4°	6°	8°	10°	12°	14°	16°

n	10	11	12	13	14	15	16	17
θ_{1n}	90°	95°	100°	105°	110°	115°	120°	125°
Φ_{1n}	18°	20°	22°	24°	25°	26°	27°	28°

Designed Mechanism:

	M	A	B	C	D
X-Coord.	0.00000	0.21583	0.09452	0.81584	1.17718
Y-Coord.	0.00000	0.18165	0.45231	1.00063	0.37948

	E	F	G	H	Q
X-Coord.	1.25503	0.65059	0.55715	0.50248	1.00000
Y-Coord.	0.39703	0.69102	0.73319	0.67281	0.00000



The steps (1), (2) and (3) of the problem 1 are common even to this problem. The step (4) in this problem is, obtaining the design equations from the constant length conditions of links BC, FE, FP, EP, HG, HS, and GS as,

$$(\bar{u}_n - \bar{v}_n)^T (\bar{u}_n - \bar{v}_n) - (\bar{u}_1 - \bar{v}_1)^T (\bar{u}_1 - \bar{v}_1) = 0 \qquad (4\text{-}10)$$

where $n = 2, 3$, etc.; and $\bar{u}$, $\bar{v}$ take the values as

$$(\bar{u}, \bar{v}) = (\bar{b}, \bar{c}), (\bar{f}, \bar{e}), (\bar{f}, \bar{p}), (\bar{e}, \bar{p}), (\bar{h}, \bar{g}), (\bar{h}, \bar{s}), (\bar{g}, \bar{s}).$$

Equations (4-10) represent a system of $6(n - 1)$ design equations involving $20 + 4(n - 1)$ unknown parameters. Hence the number of parameters to be specified is $3(7 - n)$. Hence a maximum of 7 precision positions for each of the two coupler points may be specified without specifying any unknown parameters.

An illustrative design problem of synthesizing an eight-link mechanism of Figure 2 is presented in Table XII, which shows the design requirements and the results of the synthesis.

TABLE XII

DESIGNED MECHANISM FOR GUIDANCE
OF TWO COUPLER POINTS

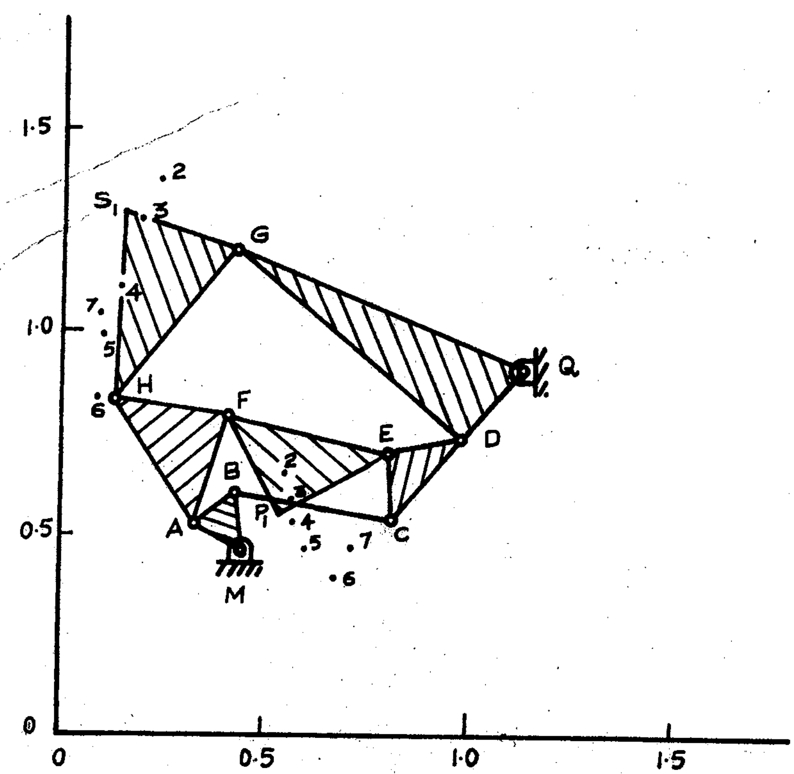
Design Specifications:

n	1	2	3	4	5	6	7
X_{pn}	0.537	0.543	0.566	0.567	0.600	0.677	0.710
Y_{pn}	0.553	0.656	0.599	0.533	0.477	0.393	0.467
X_{sn}	0.153	0.237	0.193	0.143	0.107	0.090	0.093
Y_{sn}	1.290	1.373	1.270	1.143	0.997	0.836	1.050

Designed Mechanism:

	M	A	B	C	D
X-Coord.	0.49608	0.33369	0.42943	0.81459	0.98416
Y-Coord.	0.45846	0.52557	0.59926	0.37169	0.73987

	E	F	G	H	Q
X-Coord.	0.74048	0.41178	0.43597	0.13151	1.13330
Y-Coord.	0.69875	0.79634	1.20008	0.83588	0.91000



CHAPTER V

APPLICATION OF THE PRINCIPLE OF LINEAR SUPERPOSITION IN THE DIMENSIONAL SYNTHESIS OF PLANAR MECHANISMS

The problem of simultaneous generation of two rigid body motions by the coupler links of an eight-link mechanism can be reduced to one simple class of problems. That is, to find sets of points (one point on each of the two rigid bodies to be guided) which retain a constant distance between the two points of the set as the two rigid bodies move through the specified positions in a plane. Figure 9 shows two rigid bodies with two such sets (AB, CD) of "special points." The number of sets of points depends upon the number and nature of positions of the two rigid bodies. For five positions there exist, in general, four sets of these points which can be located in a closed form by applying the principle of linear superposition.

The procedure for finding the above mentioned sets of points is described below:

Let $\bar{a}_n^T \triangleq (x_{an}\ y_{an}\ 1)$ denote the position vector of a point $A_n(x_{an}, y_{an})$ in an XY-coordinate system. Also let $[D(a_n, a_1, \alpha_{1n})]$ denote the displacement matrix associated with the planar motion of a rigid body carrying the point A. α_{1n} is the angular displacement of

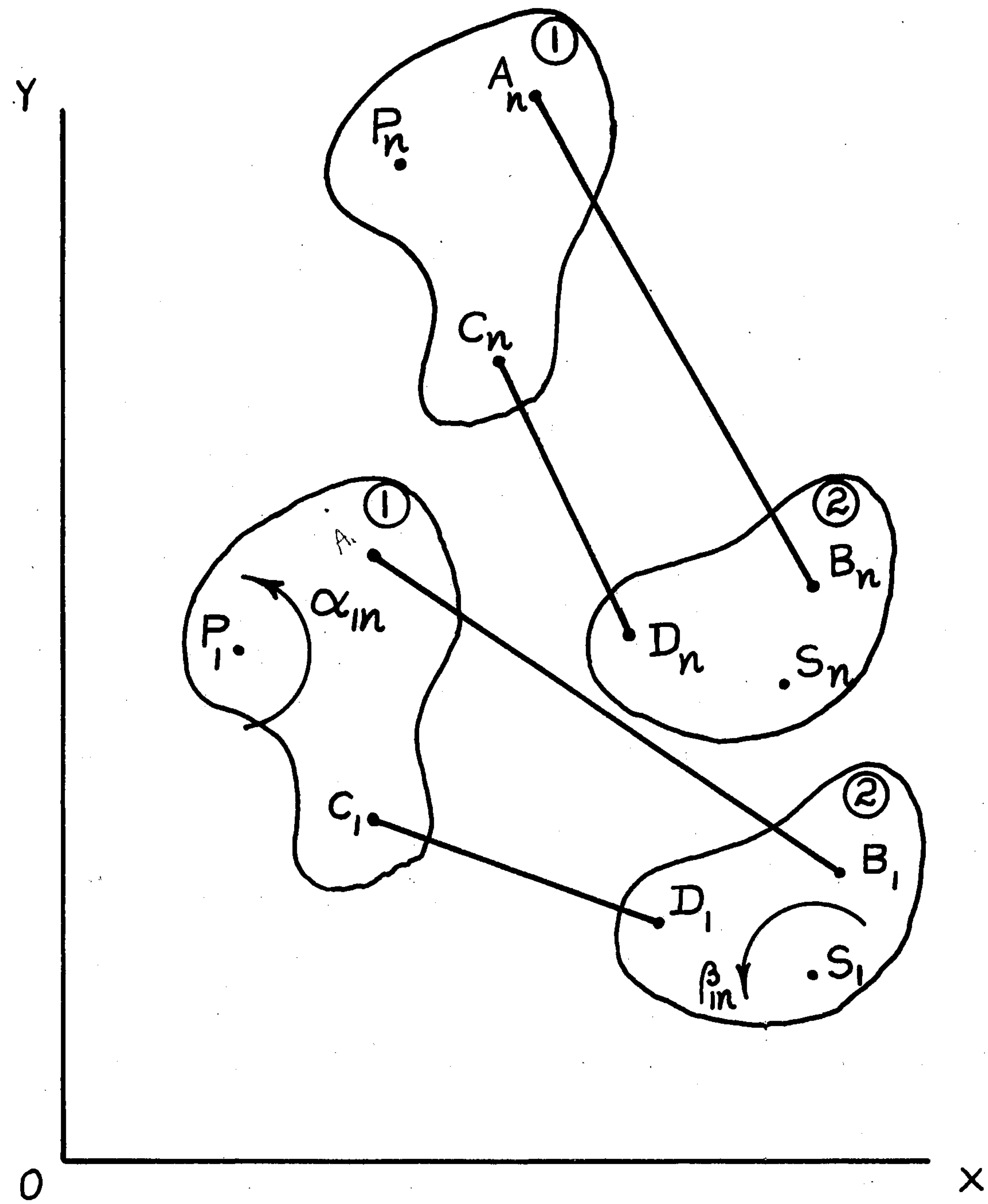


Figure 9. Special Points on Rigid Bodies

the rigid body when the point A moves from its first to the n^{th} position. Then, as described in [49], $[D(\bar{a}_n, \bar{a}_1, \alpha_{1n})]$ is given by,

$$[D(a_n, a_1, \alpha_{1n})] = \begin{bmatrix} \cos\alpha_{1n} & -\sin\alpha_{1n} & X_{an} - X_{a1}\cos\alpha_{1n} + Y_{a1}\sin\alpha_{1n} \\ \sin\alpha_{1n} & \cos\alpha_{1n} & Y_{an} - X_{a1}\sin\alpha_{1n} - Y_{a1}\cos\alpha_{1n} \\ 0 & 0 & 1 \end{bmatrix} \tag{5-1}$$

where n = 1, 2, ... Note that for n = 1, $\alpha_{1n} = 0$, $\bar{a}_n = \bar{a}_1$ and [D] becomes the identity matrix.

Let A and B be the special points in a set. The coordinates of these points may be determined as follows:

(1) The motion of the two rigid bodies is given by specifying all the positions of points P and S, and the associated angular displacements α_{1n}, β_{1n}. Then the n^{th} positions of the points A and B can be expressed in terms of their first positions by making use of the displacement matrices. Hence

$$\bar{a}_n = [D(\bar{p}_n, \bar{p}_1, \alpha_{1n})]\,\bar{a}_1$$
$$\bar{b}_n = [D(\bar{q}_n, \bar{q}_1, \beta_{1n})]\,\bar{b}_1 \tag{5-2}$$

where n = 1, ..., 5 and the displacement matrices carry the definition of equation (5-1).

(2) Since A and B are the special points so the distance between them remains constant during the motion. This condition can be mathematically expressed as,

$$(\bar{a}_n - \bar{b}_n)^T (\bar{a}_n - \bar{b}_n) = (\bar{a}_1 - \bar{b}_1)^T (\bar{a}_1 - \bar{b}_1) \qquad (5\text{-}3)$$

where n = 1, ..., 5.

(3) Substituting for $\bar{a}_n$, $\bar{b}_n$ from equation (5-2) into equation (5-3) and simplifying the resultant, we get

$$\sum_{i=1}^{5} C_{ni}\, k_i = C_{n6}\, k_6 + C_{n7}\, k_7 + C_{n8} \qquad (5\text{-}4)$$

where

$$C_{n1} = -2(X_{psn} \sin \beta_{1n} - Y_{psn} \cos \beta_{1n})$$

$$C_{n2} = 2(X_{psn} \cos \beta_{1n} + Y_{psn} \sin \beta_{1n})$$

$$C_{n3} = 2(X_{psn} \cos \alpha_{1n} + Y_{psn} \sin \alpha_{1n})$$

$$C_{n4} = -2(X_{psn} \sin \alpha_{1n} - Y_{psn} \cos \alpha_{1n})$$

$$C_{n5} = 1$$

$$C_{n6} = 2 \cos (\alpha_{1n} - \beta_{1n})$$

$$C_{n7} = 2 \sin (\alpha_{1n} - \beta_{1n})$$

$$C_{n8} = X^2_{psn} + Y^2_{psn}$$

$$X_{psn} = X_{pn} - X_{sn}$$

$$Y_{psn} = Y_{pn} - Y_{sn}$$

$$k_1 = Y_{b1} - Y_{s1}$$

$$k_2 = X_{b1} - X_{s1}$$

$$k_3 = X_{p1} - X_{a1}$$

$$k_4 = Y_{p1} - Y_{a1}$$

$$k_5 = (X_{a1} - X_{b1})^2 + (Y_{a1} - Y_{b1})^2$$
$$- (k_1^2 + k_2^2 + k_3^2 + k_4^2)$$

$$k_6 = k_1 k_4 + k_2 k_3 \tag{5-5}$$

$$k_7 = k_1 k_3 - k_2 k_4$$

$$n = 1, \ldots, 5.$$

Equation (5-4) represents a set of five equations involving seven unknown parameters, k's. However these parameters are not all independent.

Let $k_6 = \lambda_1$, $k_7 = \lambda_2$ then due to the principle of linear superposition let us define

$$k_i = L_i + \lambda_1 M_i + \lambda_2 N_i \tag{5-6}$$

where L_i, M_i, N_i ($i = 1, \ldots, 5$) are new unknown parameters. Substituting for k_i from equation (5-6) into equation (5-4) and equating the coefficients of λ_1, λ_2 and the terms independent of λ's gives

$$\underset{5\times5}{[C_{ni}]}\ \underset{5\times3}{[\bar{L}\ \bar{M}\ \bar{N}]} = \underset{5\times3}{[\bar{C}_8\ \bar{C}_6\ \bar{C}_7]} \tag{5-7}$$

where n, i = 1, ..., 5 and

$$[\bar{L}\ \bar{M}\ \bar{N}] = \begin{bmatrix} L_1 & M_1 & N_1 \\ L_2 & M_2 & N_2 \\ \vdots & \vdots & \vdots \\ L_5 & M_5 & N_5 \end{bmatrix}$$

and $\bar{C}_8^T = (C_{18}\ C_{28}\ .\ .\ .\ C_{58})$ and similar definitions hold for $\bar{C}_6$, $\bar{C}_7$.

Equation (5-7) represents three sets of equations (five linear equations in five unknown parameters in each set) in which the coefficients are functions of the known quantities. Hence $\bar{L}$, $\bar{M}$, and $\bar{N}$ can be computed from equation (5-7). The k's are then given by equation (5-6). For the systems of equations to be compatible, equations (5-5) have to be satisfied. Hence substituting from equation (5-6) in equation (5-5) and substituting $k_6 = \lambda_1$, $k_7 = \lambda_2$ gives

$$\left.\begin{aligned} \lambda_2^2 + (Z_1)\lambda_2 + (Z_2) &= 0 \\ \lambda_2^2 + (Z_3)\lambda_2 + (Z_4) &= 0 \end{aligned}\right\} \qquad (5\text{-}8)$$

where Z's are functions of λ_1 and the other known parameters.

Applying Sylvesters dialytic eliminant technique to equations (5-8) for a common root of λ_2, and simplifying we get,

$$\sum_{i=0}^{4} f_i \lambda_1^i = 0 \qquad (5\text{-}9)$$

where f's are functions of known parameters.

The real roots of λ_1 from equation (5-9) when substituted in equations (5-8) will yield corresponding roots (common for the system (5-8)) for λ_2. Equation (5-6) then gives the k's. From the definition of k's the positions of the special points A and B may be obtained as

$$X_{a1} = X_{p1} - k_3 \qquad X_{b1} = X_{s1} + k_2$$

$$Y_{a1} = Y_{p1} - k_4 \qquad Y_{b1} = Y_{s1} + k_1.$$

CHAPTER VI

SYNTHESIS OF EIGHT-LINK MECHANISMS FOR SIMULTANEOUS GUIDANCE OF TWO RIGID BODIES

This chapter presents a treatment of Design Problem No. 9. These are two types of problems considered here. There are classified on basis of the motion of the rigid bodies, that is whether it is rectilinear or otherwise.

Non-Rectilinear Motion Generation

Figure 10 illustrates the various positions of two rigid bodies executing non-rectilinear motions.

Case 1. Synthesis of Hain's Eight-Link Mechanism

Figure 11 shows the eight-link mechanism to be synthesized for motion generation of two rigid bodies. The coupler links CE and DH carry the rigid bodies to be guided. As pointed out earlier, this problem can be divided into five problems of exactly the same type as the two rigid bodies guidance problem discussed in the previous section. These problems are identified below in the synthesis procedure which involves the following steps:

(1) The motions of the two rigid bodies PCE and SDH are specified. As shown in Figure 12 (a) the problem of finding points C and D

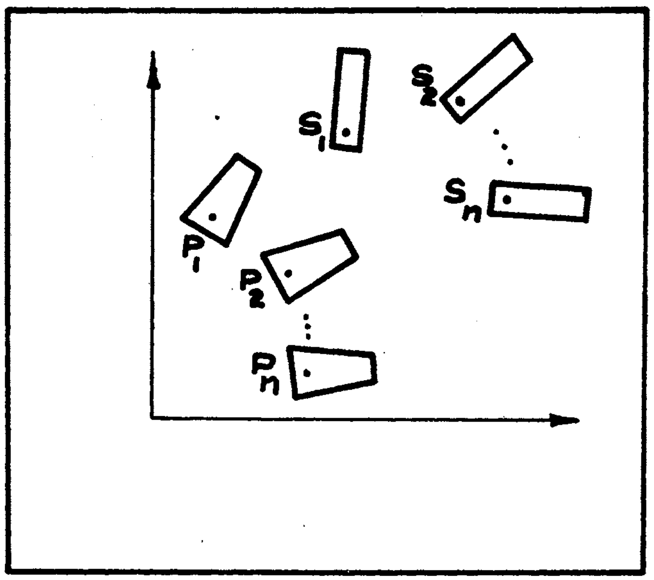


Figure 10. Non-Rectilinear Paths of Rigid Bodies

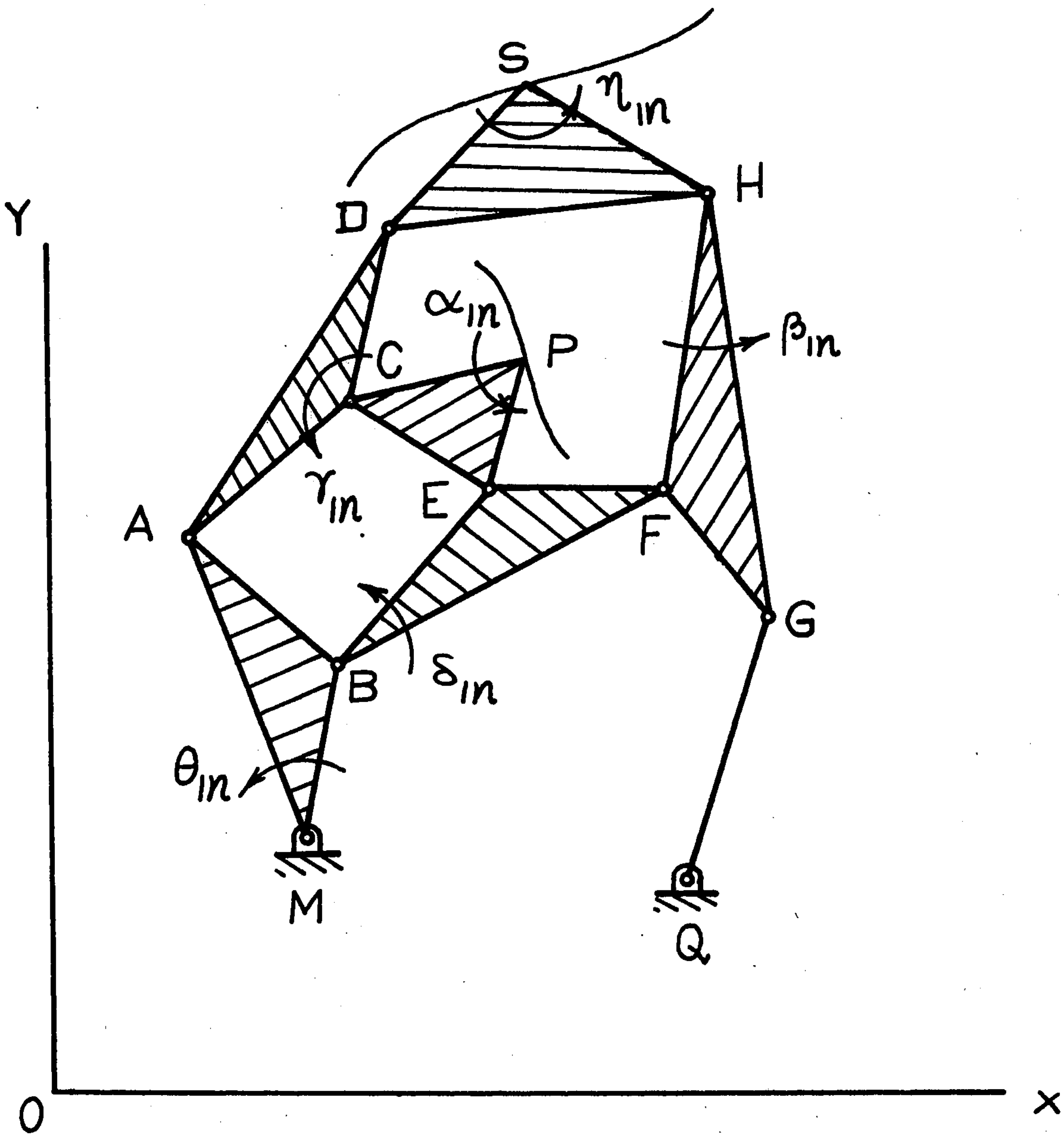


Figure 11. Hain's Eight-Link Mechanism

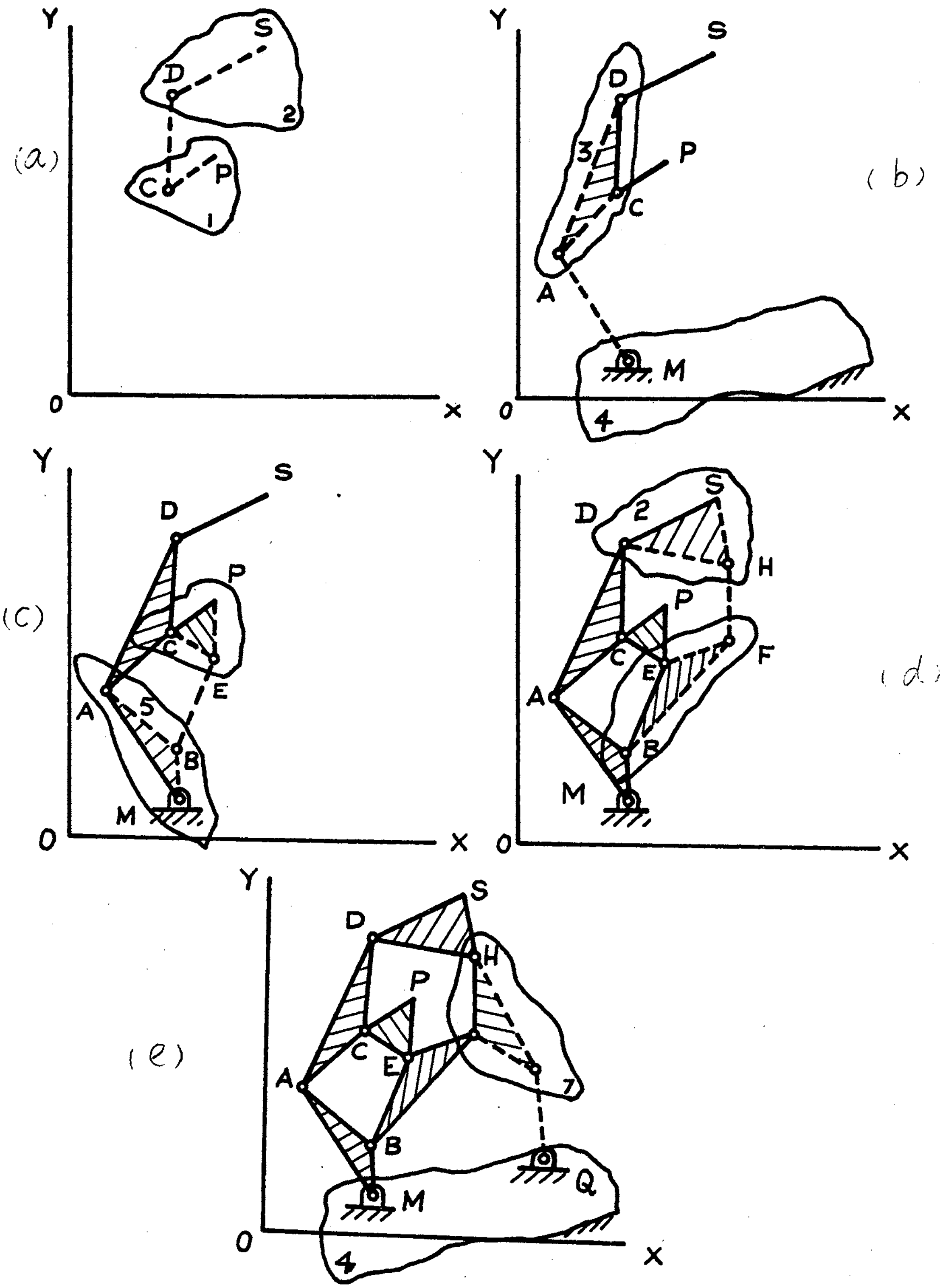


Figure 12. Steps for Synthesizing Hain's Mechanism

is the same as the one of finding points A and B in the Chapter V (Figure (9)). Hence the positions of C and D can be easily computed. There may exist a maximum of 4 sets of values for the positions of C and D.

(2) Since the first positions of C and D, i.e., $\bar{c}_1$ and $\bar{d}_1$ and the displacement matrices associated with the motion of links PC and SD are now known, the n^{th} positions of C and D are obtained from

$$\bar{c}_n = [D(\bar{p}_n, \bar{p}_1, \alpha_{1n})]\ \bar{c}_1 \tag{6-1}$$

$$\bar{d}_n = [D(\bar{s}_n, \bar{s}_1, \eta_{1n})]\ \bar{d}_1 \tag{6-2}$$

for n = 1, ..., 5. The angle γ_{1n} is given by

$$\gamma_{1n} = \tan^{-1}\left(\frac{Y_{cn} - Y_{dn}}{X_{cn} - X_{dn}}\right) - \tan^{-1}\left(\frac{Y_{c1} - Y_{d1}}{X_{c1} - X_{d1}}\right) \tag{6-3}$$

for n = 1, ..., 5.

(3) Since the motion of link CD is completely known, the next problem is to find the special points A and M for guiding the bodies 3 and 4 as shown in Figure 12 (b). Thus, this problem is of the same type as described in the step (1). Hence, using the procedure developed, a maximum of 4 sets of values, for the positions of A and M may be obtained for each set of values of positions of C and D. Thus, there may exist a maximum of 16 solutions.

(4) The analysis procedure of step (2) may be repeated again to find the rotation θ_{1n} as

$$\theta_{1n} = \tan^{-1}\left(\frac{Y_m - Y_{an}}{X_m - X_{an}}\right) - \tan^{-1}\left(\frac{Y_m - Y_{a1}}{X_m - X_{a1}}\right)$$

where the coordinates of A in the n^{th} position are given by

$$\bar{a}_n = [D(\bar{c}_n, \bar{c}_1, \gamma_{1n})]\,\bar{a}_1$$

for $n = 1, \ldots, 5$.

(5) Knowing the motion of links 5 and 1 in Figure 12 (c) the first positions of points B and E can be obtained in the same manner as described in step (1). There may exist 4 solutions of B and E for every set of the 16 solutions obtained in step (3). Thus, there may exist a maximum of 64 solutions.

Now the analysis may be carried on to calculate

$$\delta_{1n} = \tan^{-1}\left(\frac{Y_{bn} - Y_{en}}{X_{bn} - X_{en}}\right) - \tan^{-1}\left(\frac{Y_{b1} - Y_{e1}}{X_{b1} - X_{e1}}\right)$$

where the positions of B and E are given by

$$\bar{b}_n = [D(\bar{m}, \bar{m}, \theta_{1n})]\,\bar{b}_1$$

$$\bar{e}_n = [D(\bar{c}_n, \bar{c}_1, \alpha_{1n})]\,\bar{e}_1$$

for $n = 1, \ldots, 5$.

(6) As shown in Figure 12 (d), the points F and H may be located on links 6 and 2 since we know the motion of these two links. Clearly, we observe that there may exist 4 solutions for each of the 64 solutions obtained in step (5). Thus there may exist a maximum of 256 solutions. The analysis then gives the angular displacements as

$$\beta_{1n} = \tan^{-1} \left(\frac{Y_{fn} - Y_{hn}}{X_{fn} - X_{hn}} \right) - \tan^{-1} \left(\frac{Y_{f1} - Y_{h1}}{X_{f1} - X_{h1}} \right)$$

where the positions of F and H are given by

$$\bar{f}_n = [D(\bar{b}_n, \bar{b}_1, \delta_{1n})] \, \bar{f}_1$$

$$\bar{h}_n = [D(\bar{d}_n, \bar{d}_1, \eta_{1n})] \, \bar{h}_1$$

where $n = 1, \ldots, 5$.

(7) Now the motion of link 7, in Figure 12 (e), is known. And the link 4 is stationary. So the points G and Q may be located in the plane of links 4 and 7. There may exist 4 solutions for each of the 256 solutions of step (6). Thus there may exist a maximum of 1024 solutions.

This completes the synthesis procedure yielding a one-degree of freedom linkage. Table XIII shows the example problem with the solutions.

Case 2. Synthesis of Eight-Link Mechanism (with all loops of the mechanism containing five links)

Figure 2 shows the eight-link mechanism with five links in each loop. The rigid bodies, to be guided, are located on links HG and CE.

The synthesis procedure is similar to the one described in the previous sections and is schematically illustrated in Figure 13. The following steps lead to the final design:

TABLE XIII

DESIGN SPECIFICATIONS AND THE DESIGNED MECHANISMS FOR SIMULTANEOUS GUIDANCE OF TWO RIGID BODIES (NON-RECTILINEAR) USING HAIN'S 8-LINK MECHANISM (FIG. 11)

DESIGN SPECIFICATIONS

PX	PY	ALFA in degrees	SX	SY	ETA in degrees
3.50000	0.0	0.0	-3.00000	2.50000	0.0
3.06074	0.70743	12.00000	-3.70743	2.93926	-12.00000
2.00000	1.00000	24.00000	-4.00000	4.00000	-24.00000
0.93926	0.70743	36.00000	-3.70743	5.06074	-36.00000
0.50000	0.0	48.00000	-3.00000	5.50000	-48.00000

A	B	C	D	E	MECHANISM	PARAMETERS				
1					C(	1.2761459,	1.2192926)	D(	3.1906919,	0.8792028)
	1				A(	1.2113590,	0.6412295)	M(	0.1095960,	-0.8453444)
		1			B(	1.2113867,	0.6412346)	E(	1.2761726,	1.2193012)
			1		F(	3.1907539,	0.8792673)	H(	-0.5323076,	2.6966906)
				1	G(	0.8838730,	1.5021067)	Q(	0.0163544,	2.3897886)
				2	G(	0.2463856,	3.9452276)	Q(	-0.0979052,	3.9998941)
				3	G(	-1.1427145,	6.0539322)	Q(	0.0164213,	5.6101685)
				4	G(	31.9329071,	18.0550385)	Q(	-9.7359362,	3.9995575)
			2		F(	2.4600410,	1.7371149)	H(	3.1916475,	0.8781319)
				1	G(	1.2096863,	0.6403980)	Q(	0.1072927,	-0.8464358)
				2	G(	3.4753065,	-0.8893805)	Q(	-1.4492426,	11.6941566)

TABLE XIII (continued)

		2			B(	1.4098701,	0.6273499)	E(	1.6574507,	1.3938274)
			1		F(	4.0293179,	0.9427746)	H(	3.8590260,	1.1276731)
				1	G(	4.0919542,	1.5147924)	Q(	-5.8305311,	16.0290375)
				2	G(	5.0671530,	1.0434275)	Q(	3.1419487,	-5.4898720)
			2		F(	3.5496340,	0.8906633)	H(	3.4841967,	1.1210537)
				1	G(	1.4241152,	2.2062454)	Q(	0.8536595,	-0.3546674)
				2	G(	3.7697058,	1.3621273)	Q(	-5.3144598,	14.6404743)
	2				A(	3.4722452,	-0.8836851)	M(	-1.4615698,	11.7425251)
		1			B(	3.4722538,	-0.8836871)	E(	1.2761593,	1.2192736)
			1		F(	3.1907377,	0.8791931)	H(	4.4602900,	1.2346258)
				1	G(	0.8837881,	1.5010605)	Q(	0.0161419,	2.3891563)
				2	G(	0.2462540,	3.9448366)	Q(	-0.0980996,	3.9996090)
				3	G(	-1.1439495,	6.0544100)	Q(	0.0160456,	5.6104670)
				4	G(	31.9832001,	18.0694885)	Q(	-9.7318811,	4.0017700)
			2		F(	1.7146711,	1.6698465)	H(	3.1906490,	0.8792267)
				1	G(	1.2114048,	0.6412640)	Q(	0.1096550,	-0.8453187)
				2	G(	3.4721642,	-0.8835907)	Q(	-1.4618454,	11.7438841)
		2			B(	-0.4170065,	10.4416018)	E(	-4.4961853,	-28.4980316)
			1		F(	-1.0015316,	7.1338196)	H(	-1.4615202,	6.7361326)
				1	G(	-0.9380673,	7.2028666)	Q(	-0.1924324,	7.7862625)
				2	G(	-1.7853966,	6.4430380)	Q(	0.0787048,	6.1271362)
			2		F(	-1.0352793,	7.0037689)	H(	-1.3303471,	6.5658789)
				1	G(	-0.9962647,	7.0759153)	Q(	-0.2001753,	7.7741241)
				2	G(	-1.4538660,	6.3907604)	Q(	0.0893593,	5.9889374)
			3		F(	-1.0777636,	6.0077057)	H(	-1.0338726,	4.6946840)
				1	G(	-1.0753765,	6.1935110)	Q(	0.0729616,	8.0225334)
				2	G(	-1.0372286,	5.1610260)	Q(	-0.0456342,	4.7119560)
				3	G(	-1.9703598,	5.7954693)	Q(	1.3357735,	1.4179277)
				4	G(	-0.4374289,	4.0596256)	Q(	-0.9763864,	4.4552526)
			4		F(	-3.2205343,	6.3764038)	H(	5.9336224,	11.0559816)
					NO SOLUTION EXISTS FOR THIS STEP					
2					C(	3.6249952,	-2.9900970)	D(	-0.9207792,	2.8149366)
	1				A(	2.7556696,	-0.1644878)	M(	0.2693937,	-0.8324001)
		1			B(	2.7556171,	-0.1644966)	E(	3.6249456,	-2.9901943)
			1		F(	-2.2367411,	4.2082014)	H(	-0.9212399,	2.8153229)
				1	G(	2.7567759,	-0.1687651)	Q(	0.2811680,	-0.8265387)
				2	G(	-1.0794926,	4.9585657)	Q(	-1.6244001,	5.5456791)
				3	G(	5.6898537,	-5.7612438)	Q(	5.5238132,	-4.9414415)
			2		F(	-0.9206791,	2.8149366)	H(	-6.4841070,	1.9953470)

TABLE XIII (continued)

			1	G(	-1.1425428,	6.0540276)	Q(	0.0164516,	5.6101093)
			2	G(	0.8836250,	1.5024462)	Q(	0.0162408,	2.3900900)
			3	G(	0.2461939,	3.9456396)	Q(	-0.0979474,	4.0002146)
			4	G(	31.9300690,	18.0496063)	Q(	-9.7361155,	4.0008240)
	2			B(	4.9175615,	-1.1101656)	E(	5.8055220,	-1.9822721)
		1		F(	6.6506624,	0.4174690)	H(	-17.8230286,	7.2878637)
			1	G(	0.4505548,	-0.5467557)	Q(	10.6769247,	1.3155756)
			2	G(	-16.7061310,	13.9601631)	Q(	-3.7526751,	11.0672760)
		2		F(	3.2354774,	5.3315763)	H(	7.0031815,	7.0415602)
			1	G(	7.1044903,	7.4799261)	Q(	-6.6303282,	4.4654455)
			2	G(	3.0272436,	24.8185120)	Q(	4.0241232,	3.6355524)
	3			B(	-2.0052443,	-2.3688250)	E(	-1.0994644,	-1.5131416)
		1		F(	-1.5925035,	-2.5182304)	H(	0.4714422,	18.5541382)
				NO SOLUTION EXISTS FOR THIS STEP					
		2		F(	-1.0877457,	1.3276920)	H(	3.8516550,	-0.6880894)
			1	G(	-0.7497435,	-0.1696997)	Q(	-0.4183661,	-3.2407389)
			2	G(	3.1570311,	-0.8726349)	Q(	-1.3380375,	5.3334875)
	4			B(	1.9748774,	7.1813297)	E(	2.7108917,	-9.3941822)
		1		F(	6.1107140,	-7.8327408)	H(	-1.7995234,	3.5795918)
				NO SOLUTION EXISTS FOR THIS STEP					
		2		F(	0.9423313,	1.3638945)	H(	1.5754385,	1.4642906)
			1	G(	0.4720910,	1.3439322)	Q(	-0.4209468,	0.8375922)
			2	G(	1.4417601,	1.4970798)	Q(	0.3419800,	2.8060932)
2				A(	5.6943521,	-5.7466717)	M(	5.5255613,	-4.9279823)
	1			B(	5.6943502,	-5.7466097)	E(	3.6249762,	-2.9905891)
				NO SOLUTION EXISTS FOR THIS STEP					
	2			B(	5.6269608,	-5.5397873)	E(	3.6957045,	-4.4049597)
		1		F(	-0.0297565,	-2.2724895)	H(	-2.3077784,	3.2137766)
				NO SOLUTION EXISTS FOR THIS STEP					
		2		F(	-12.1435080,	12.2850885)	H(	-16.8912201,	13.8125648)
			1	G(	-13.6718283,	12.7745094)	Q(	-0.0171419,	5.6040878)
			2	G(	-17.8498688,	12.8258991)	Q(	-4.7691078,	9.4858980)
	3			B(	4.4723625,	-5.9489145)	E(	2.9316368,	-9.0623798)
		1		F(	7.0115824,	-10.7404022)	H(	4.7384968,	-0.4655552)
			1	G(	4.0927267,	-5.1846991)	Q(	0.5762182,	-4.5978184)
			2	G(	2.9466906,	-3.5007658)	Q(	-5.2126722,	-0.1679583)
		2		F(	2.4424734,	-7.5521021)	H(	-1.3459940,	5.1534863)
			1	G(	-0.1387339,	4.2723656)	Q(	0.7415810,	4.6139946)
			2	G(	-9.1448221,	10.4991827)	Q(	-8.7752924,	9.1940546)

TABLE XIII (continued)

	4			B(	7.5373287,	-2.7426748)	E(	6.0197554,	-7.1990204)
		1		F(	6.9436064,	-7.3680878)	H(	-2.5037575,	0.9301739)
			1	G(	-2.4586267,	3.2825193)	Q(	-2.8633080,	5.0650768)
		2		F(	4.6080046,	-6.2576618)	H(	4.4539022,	1.9236317)
			1	G(	4.2732182,	-1.0027380)	Q(	0.5996245,	-0.8566092)
			2	G(	4.5212793,	-1.4239769)	Q(	3.9036827,	-1.8485937)
		3		F(	8.0531530,	-3.3292513)	H(	8.5561419,	0.6310101)
			1	G(	7.7466936,	-3.4728241)	Q(	5.3237476,	-4.8039789)
			2	G(	7.6896210,	-2.1137924)	Q(	6.0449057,	-4.2123833)
			3	G(	7.3834982,	0.0906553)	Q(	10.1943665,	-6.5138855)
			4	G(	8.8307743,	0.2234526)	Q(	-10.0075073,	15.7307892)
		4		F(	12.3150740,	-1.6997051)	H(	7.3607941,	-1.0434847)
			1	G(	10.0022850,	-2.0794525)	Q(	5.7786627,	-5.0539417)
			2	G(	7.6398573,	-0.8561535)	Q(	-10.0273438,	19.8707581)
3				A(	-1.0785599,	4.9578657)	M(	-1.6233339,	5.5449381)
	1			B(	-1.0781460,	4.9576874)	E(	3.6256676,	-2.9872541)
		1		F(	-1.1053524,	2.6570749)	H(	-0.9231052,	2.8096552)
			1	G(	2.7266722,	-0.2912607)	Q(	0.5887779,	-0.6465912)
			2	G(	-1.1477699,	4.9678926)	Q(	-1.7105560,	5.5489235)
			3	G(	5.4476414,	-6.2164755)	Q(	5.3867407,	-5.3576965)
		2		F(	-0.9186592,	2.8156738)	H(	-0.2462578,	4.3773670)
			1	G(	-1.1400375,	6.0496016)	Q(	0.0176880,	5.6060438)
			2	G(	0.8843260,	1.5023298)	Q(	0.0153753,	2.3924236)
			3	G(	0.2455912,	3.9466906)	Q(	-0.0985844,	4.0011940)
			4	G(	31.5800018,	17.9711456)	Q(	-9.7233620,	3.9887238)
	2			B(	-1.1518030,	4.9942350)	E(	3.5008421,	-3.5657110)
		1		F(	-1.2305393,	2.7972975)	H(	-2.4385099,	-0.0004082)
			1	G(	-1.1214180,	5.1264477)	Q(	-0.2750200,	4.6976337)
			2	G(	0.4398651,	1.6206865)	Q(	-0.4112840,	2.4191160)
			3	G(	-0.2216530,	3.6481104)	Q(	-0.5251248,	3.6609077)
			4	G(	9.6396303,	9.1525526)	Q(	-10.1373701,	3.3668795)
		2		F(	-0.8040323,	2.9433899)	H(	-0.7477064,	2.9848652)
			1	G(	-1.4644766,	3.3885250)	Q(	0.0445455,	-1.1083555)
	3			B(	-3.5103121,	6.0193052)	E(	0.1851444,	2.4360008)
		1		F(	1.6516476,	0.8509455)	H(	1.9696455,	-1.6665325)
			1	G(	0.5287838,	-1.2967243)	Q(	-5.1812258,	9.2918072)
			2	G(	15.8375063,	-25.8816223)	Q(	-4.2278519,	-3.0451107)
		2		F(	-0.6827867,	2.7424374)	H(	4.3969727,	2.0900898)
			1	G(	0.8999331,	3.4546328)	Q(	1.5664787,	-0.1174959)
			2	G(	30.4538422,	-29.5575104)	Q(	-2.4918823,	9.1910410)
		3		F(	-5.0611877,	6.7325010)	H(	-2.6236620,	6.2348118)

			1	G(	-3.0627298,	6.6838245)	Q(	0.8381443,	-3.2250500)
			2	G(	-2.3366747,	6.4375868)	Q(	-0.6112552,	6.3702869)
		4		F(	-4.2152920,	0.3171043)	H(	146.3839569,	42.4461517)
			1	G(	-106.1471252,	-40.6815643)	Q(	-1.4915037,	4.4804688)
			2	G(	61.0322571,	10.5730553)	Q(	-15.5669069,	8.7189178)
	4			B(	-1.4910917,	5.5371513)	E(	1.1686859,	-69.4902496)
		1		F(	-0.7894783,	5.3814545)	H(	-2.4416637,	6.0247335)
			1	G(	-0.6537198,	5.9543791)	Q(	-0.5862691,	5.8513432)
			2	G(	-0.4943805,	5.7850227)	Q(	-1.1596022,	6.5672503)
		2		F(	-1.4715872,	3.7310486)	H(	-1.1849127,	1.7063427)
			1	G(	-2.0585642,	3.4549332)	Q(	2.1612291,	1.0296707)
			2	G(	-1.3370991,	3.3418760)	Q(	-0.3388643,	3.2336264)
			3	G(	-1.8395853,	4.0312777)	Q(	-1.9489040,	6.3796110)
			4	G(	-0.6047192,	2.8900366)	Q(	-1.0269365,	2.8871126)
		3		F(	-1.2366152,	4.5578461)	H(	-1.0379848,	4.1911221)
			1	G(	-1.3443260,	4.4909449)	Q(	0.4757272,	3.0527306)
			2	G(	-1.3359404,	4.5857487)	Q(	-1.3032913,	4.9905367)
			3	G(	-1.1861038,	4.5290356)	Q(	-0.5070719,	4.5298672)
			4	G(	-1.5350714,	4.5882072)	Q(	-1.0044060,	4.1415195)
		4		F(	-3.4480953,	5.0401154)	H(	16.1328735,	12.7126579)
			1	G(	1.1521244,	2.6981926)	Q(	-8.0416737,	5.2951536)
			2	G(	-7.3157015,	5.3171892)	Q(	-4.2359028,	2.9861937)
4				A(	19.2303467,	9.8001404)	M(	-65.0642700,	-43.5281372)
	1			B(	19.2919312,	9.8579884)	E(	3.6283464,	-2.9886456)
				NO SOLUTION EXISTS FOR THIS STEP					
	2			B(	46.9464111,	36.3122253)	E(	4.3635321,	-2.6440315)
		1		F(	-4.5245600,	6.6216354)	H(	-0.8986320,	1.8180389)
			1	G(	10.9455519,	-4.6135511)	Q(	9.9612665,	-4.3079472)
			2	G(	-17.0769043,	41.1614838)	Q(	-16.7365417,	42.2584839)
			3	G(	-22.1628418,	17.7471313)	Q(	-24.2972565,	18.5304260)
			4	G(	29.4129028,	32.7030640)	Q(	33.3316956,	33.4600372)
		2		F(	15.4449635,	-28.7043152)	H(	-1.5268469,	2.3134632)
			1	G(	5.3941841,	-3.2809448)	Q(	2.5374889,	-3.7962999)
			2	G(	-1.4741497,	6.0018768)	Q(	-1.9433413,	6.9263401)
			3	G(	10.0416327,	-12.1848450)	Q(	9.6136045,	-11.2410660)

TOTAL NUMBER OF MECHANISMS= 93

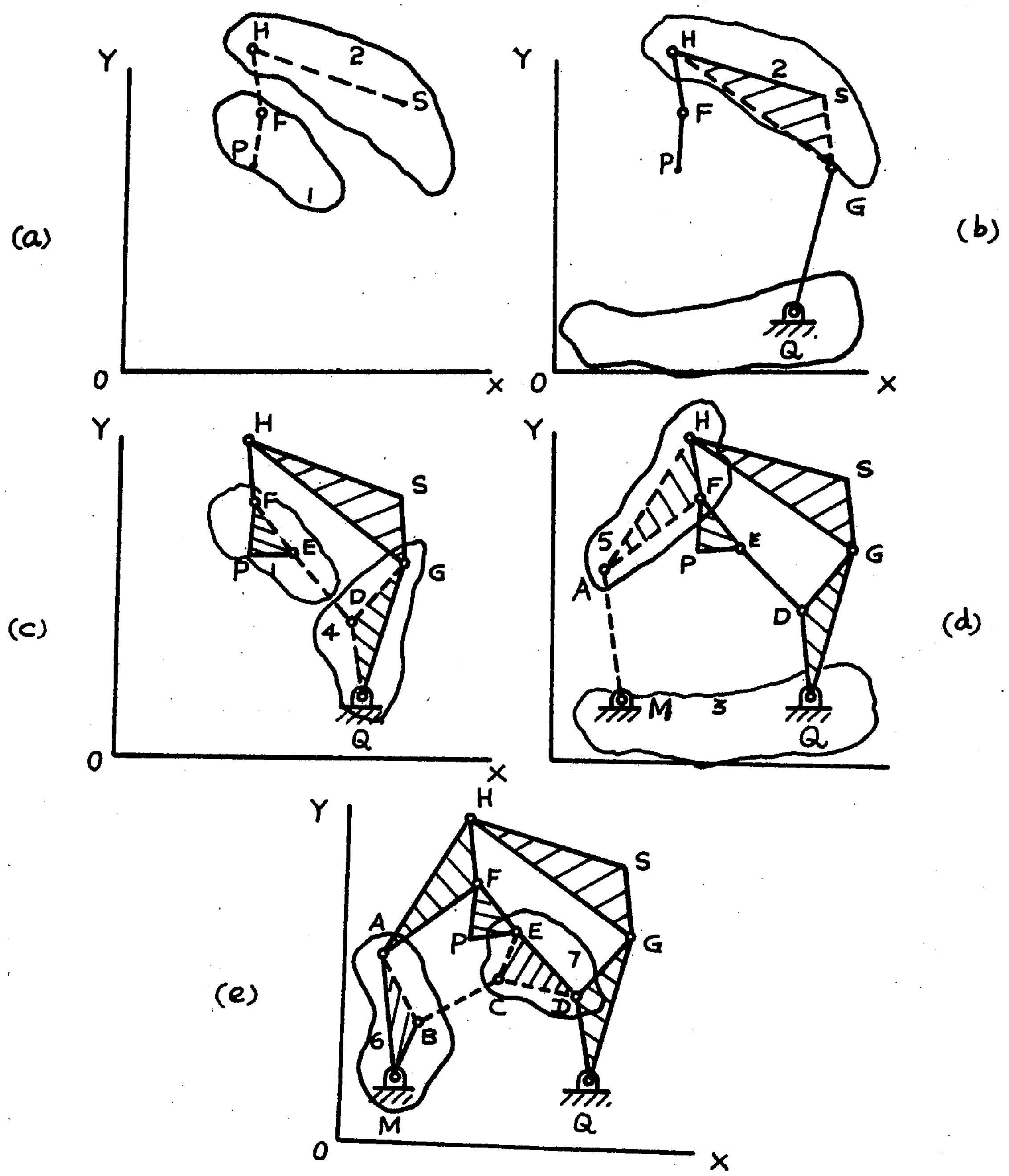


Figure 13. Steps for Synthesizing Eight-Link Mechanism

(1) The motion of the two rigid bodies 1 and 2, shown in Figure 13 (a), is known. The points F and H can be determined in the planes of the rigid bodies 1 and 2. Having located F and H, the angular displacements γ_{1n} of link HF can be found from

$$\gamma_{1n} = \tan^{-1} \left(\frac{Y_{fn} - Y_{hn}}{X_{fn} - X_{hn}} \right) - \tan^{-1} \left(\frac{Y_{f1} - Y_{h1}}{X_{f1} - X_{h1}} \right)$$

where the n^{th} positions of F and H are given by

$$\bar{f}_n = [D(\bar{p}_n, \bar{p}_1, \alpha_{1n})] \bar{f}_1$$

$$\bar{h}_n = [D(\bar{s}_n, \bar{s}_1, \eta_{1n})] \bar{h}_1$$

for $n = 1, \ldots, 5$.

(2) Since motion of link 2 is known and link 3 (in Figure 13 (b)) is stationary, the points Q and G can be located. Then the rotation angles ϕ_{1n} of link QG may be computed as

$$\phi_{1n} = \tan^{-1} \left(\frac{Y_q - Y_{gn}}{X_q - X_{gn}} \right) - \tan^{-1} \left(\frac{Y_q - Y_{g1}}{X_q - X_{g1}} \right)$$

where the n^{th} position of G is given by

$$\bar{g}_n = [D(\bar{s}_n, \bar{s}_1, \eta_{1n})] \bar{g}_1$$

for $n = 1, \ldots, 5$.

(3) Knowing the motion of links 1 and 4 in Figure 13 (b), the points D and E can be easily located. The angular displacements β_{1n} of link DE, required in the subsequent synthesis, are given as

$$\beta_{1n} = \tan^{-1} \left(\frac{Y_{dn} - Y_{en}}{X_{dn} - X_{en}} \right) - \tan^{-1} \left(\frac{Y_{d1} - Y_{e1}}{X_{d1} - X_{e1}} \right)$$

where the n^{th} positions of D and E are given by

$$\bar{d}_n = [D(\bar{q}, \bar{q}, \Phi_{1n})]\, \bar{d}_1$$

$$\bar{e}_n = [D(\bar{p}_n, \bar{p}_1, \alpha_{1n})]\, \bar{e}_1$$

for n = 1, ..., 5.

(4) The points M and A on links 3 and 5 in Figure 13 (d) can now be obtained. And the angular displacements θ_{1n}, of the link AM, are given as:

$$\theta_{1n} = \tan^{-1}\left(\frac{Y_m - Y_{an}}{X_m - X_{an}}\right) - \tan^{-1}\left(\frac{Y_m - Y_{a1}}{X_m - X_{a1}}\right)$$

where the n^{th} position of A is given by

$$\bar{a}_n = [D(\bar{m}, \bar{m}, \theta_{1n})]\, \bar{a}_1$$

for n = 1, ..., 5.

(5) The points B and C on links 6 and 7, as shown in Figure 13 (e), can now be located, thereby completing the synthesis procedure.

For every step of the synthesis procedure, there may exist 4 solutions. Since there are 5 steps involved there may exist as many as 4^5 = 1024 solutions. Table XIV shows an illustrative example of this type of synthesis problem.

Rectilinear Motion Generation

Figure 14 illustrates the rectilinear motions of two rigid bodies.

The synthesis procedures of the two preceding sections do not afford a solution to the special case of generating two rectilinear

TABLE XIV

DESIGN SPECIFICATIONS AND THE DESIGNED
MECHANISMS FOR SIMULTANEOUS GUIDANCE
OF TWO RIGID BODIES USING
EIGHT-LINK MECHANISM
(Fig. 2)

DESIGN SPECIFICATIONS

PX	PY	ALFA	SX	SY	ETA
3.50000	0.0	0.0	-3.00000	2.50000	0.0
3.06074	0.70743	12.00000	-3.70743	2.93926	-12.00000
2.00000	1.00000	24.00000	-4.00000	4.00000	-24.00000
0.93926	0.70743	36.00000	-3.70743	5.06074	-36.00000
0.50000	0.0	48.00000	-3.00000	6.50000	-48.00000

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(1.2761, -1.2193) G(X,Y)=(0.8840, 1.5014)
H(X,Y)=(3.1907, 0.8792) O(X,Y)=(0.0164, 2.3893)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.110, -0.845)	(1.211, 0.641)	(0.282, 0.532)	(0.714, 0.062)	(5.251, -2.468)	(0.819, 0.262)
(0.110, -0.845)	(1.211, 0.641)	(1.005, 0.201)	(0.802, -0.186)	(5.251, -2.468)	(0.819, 0.262)
(-1.462, 11.743)	(3.472, -0.884)	(0.774, -0.234)	(1.159, -0.896)	(5.251, -2.468)	(0.819, 0.262)
(-1.462, 11.743)	(3.472, -0.884)	(2.329, -1.223)	(1.121, 0.003)	(5.251, -2.468)	(0.819, 0.262)
(-1.462, 11.743)	(3.472, -0.884)	(-6.570, -0.409)	(0.426, -1.621)	(5.251, -2.468)	(0.819, 0.262)
(-1.462, 11.743)	(3.472, -0.884)	(4.303, 3.082)	(2.109, -2.911)	(5.251, -2.468)	(0.819, 0.262)
(0.110, -0.845)	(1.211, 0.641)	(-0.543, -1.272)	(-0.103, 1.285)	(0.281, 2.566)	(3.687, 8.217)
(0.110, -0.845)	(1.211, 0.641)	(-20.157, 0.949)	(-10.733, -3.799)	(0.281, 2.566)	(3.687, 8.217)
(-1.462, 11.743)	(3.472, -0.884)	(-0.548, 12.560)	(0.534, 2.559)	(0.281, 2.566)	(3.687, 8.217)
(-1.462, 11.743)	(3.472, -0.884)	(3.377, 0.763)	(1.163, 3.948)	(0.281, 2.566)	(3.687, 8.217)

TABLE XIV (continued)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(1.2761, 1.2193) G(X,Y)=(-1.1424, 6.0538)
H(X,Y)=(3.1907, 0.8792) Q(X,Y)=(0.0163, 5.6100)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.110, -0.845)	(1.211, 0.641)	(0.427, -0.750)	(0.962, 2.736)	(0.174, 6.285)	(-0.727, -6.577)
(0.110, -0.845)	(1.211, 0.641)	(37.243, -1.794)	(-1.994, -2.413)	(0.174, 6.285)	(-0.727, -6.577)
(0.110, -0.845)	(1.211, 0.641)	(0.303, -1.063)	(0.861, 6.119)	(-0.144, 5.697)	(-2.221, -23.111)
(0.110, -0.845)	(1.211, 0.641)	(-74.001, -25.099)	(-3.048, -4.651)	(-0.144, 5.697)	(-2.221, -23.111)
(-1.462, 11.743)	(3.472, -0.884)	(-0.417, 9.956)	(1.420, 14.996)	(-0.144, 5.697)	(-2.221, -23.111)
(-1.462, 11.743)	(3.472, -0.884)	(0.180, 7.534)	(-0.395, 8.400)	(-0.144, 5.697)	(-2.221, -23.111)
(0.110, -0.845)	(1.211, 0.641)	(1.125, 0.579)	(1.473, 2.177)	(2.207, 5.849)	(1.558, 1.915)
(0.110, -0.845)	(1.211, 0.641)	(1.792, 0.519)	(2.658, 0.736)	(2.207, 5.849)	(1.558, 1.915)
(-1.462, 11.743)	(3.472, -0.884)	(9.146, -4.307)	(1.908, 2.696)	(2.207, 5.849)	(1.558, 1.915)
(-1.462, 11.743)	(3.472, -0.884)	(-15.392, 1.570)	(-12.342, -1.016)	(2.207, 5.849)	(1.558, 1.915)
(-1.462, 11.743)	(3.472, -0.884)	(-19.150, 11.768)	(-12.680, 5.890)	(2.207, 5.849)	(1.558, 1.915)
(-1.462, 11.743)	(3.472, -0.884)	(4.418, 13.148)	(2.400, 15.103)	(2.207, 5.849)	(1.558, 1.915)
(0.110, -0.845)	(1.211, 0.641)	(3.097, -0.911)	(2.458, 2.063)	(5.535, 4.540)	(4.263, 2.007)
(0.110, -0.845)	(1.211, 0.641)	(2.307, 0.297)	(4.245, 2.247)	(5.535, 4.540)	(4.263, 2.007)
(-1.462, 11.743)	(3.472, -0.884)	(3.150, 2.870)	(3.936, 1.652)	(5.535, 4.540)	(4.263, 2.007)
(-1.462, 11.743)	(3.472, -0.884)	(-0.326, 2.345)	(2.829, 0.757)	(5.535, 4.540)	(4.263, 2.007)
(-1.462, 11.743)	(3.472, -0.884)	(0.736, 7.042)	(5.098, 3.481)	(5.535, 4.540)	(4.263, 2.007)
(-1.462, 11.743)	(3.472, -0.884)	(-13.086, 10.364)	(2.999, 2.844)	(5.535, 4.540)	(4.263, 2.007)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(1.2761, 1.2193) G(X,Y)=(0.2466, 3.9446)
H(X,Y)=(3.1907, 0.8792) Q(X,Y)=(-0.0979, 3.9995)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.110, -0.845)	(1.211, 0.641)	(0.006, -1.148)	(0.287, 3.581)	(-0.063, 4.051)	(11.203, 59.590)
(0.110, -0.845)	(1.211, 0.641)	(-24.286, -9.030)	(-4.720, -5.533)	(-0.063, 4.051)	(11.203, 59.590)
(-1.462, 11.743)	(3.472, -0.884)	(-0.632, 12.331)	(-0.312, 3.971)	(-0.063, 4.051)	(11.203, 59.590)
(-1.462, 11.743)	(3.472, -0.884)	(1.603, 3.564)	(0.281, 5.856)	(-0.063, 4.051)	(11.203, 59.590)
(0.110, -0.845)	(1.211, 0.641)	(-0.256, -0.430)	(0.596, -1.493)	(0.027, 3.754)	(1.748, -5.294)
(0.110, -0.845)	(1.211, 0.641)	(0.623, -1.207)	(4.464, 5.869)	(0.027, 3.754)	(1.748, -5.294)
(0.110, -0.845)	(1.211, 0.641)	(2.869, -0.680)	(3.509, 0.177)	(1.269, 3.869)	(3.714, 1.118)
(0.110, -0.845)	(1.211, 0.641)	(2.431, -0.925)	(2.589, -0.178)	(1.269, 3.869)	(3.714, 1.118)
(-1.462, 11.743)	(3.472, -0.884)	(2.897, 2.804)	(3.257, 2.036)	(1.269, 3.869)	(3.714, 1.118)
(-1.462, 11.743)	(3.472, -0.884)	(2.400, 4.392)	(2.459, 3.528)	(1.269, 3.869)	(3.714, 1.118)
(0.110, -0.845)	(1.211, 0.641)	(1.766, 0.568)	(0.963, 1.633)	(-1.795, 3.664)	(1.323, 2.046)
(0.110, -0.845)	(1.211, 0.641)	(1.102, 0.417)	(1.197, 1.994)	(-1.795, 3.664)	(1.323, 2.046)
(-1.462, 11.743)	(3.472, -0.884)	(-11.277, 10.769)	(0.287, 1.911)	(-1.795, 3.664)	(1.323, 2.046)
(-1.462, 11.743)	(3.472, -0.884)	(5.501, -1.983)	(1.102, 2.178)	(-1.795, 3.664)	(1.323, 2.046)
(-1.462, 11.743)	(3.472, -0.884)	(0.051, -0.070)	(1.884, 0.535)	(-1.795, 3.664)	(1.323, 2.046)
(-1.462, 11.743)	(3.472, -0.884)	(1.501, 3.449)	(-0.215, 3.784)	(-1.795, 3.664)	(1.323, 2.046)

TABLE XIV (continued)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(1.2761, 1.2193) G(X,Y)=(31.9396, 18.0560)
H(X,Y)=(3.1907, 0.8792) Q(X,Y)=(-9.7359, 4.0002)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.110, -0.845)	(1.211, 0.641)	(-2.085, -3.997)	(-5.936, 2.011)	(-3.777, 2.884)	(-2.807, 4.422)
(0.110, -0.845)	(1.211, 0.641)	(-11.286, 8.467)	(-13.928, 11.529)	(-3.777, 2.884)	(-2.807, 4.422)
(-1.462, 11.743)	(3.472, -0.884)	(3.406, 10.467)	(-4.959, 1.902)	(-3.777, 2.884)	(-2.807, 4.422)
(-1.462, 11.743)	(3.472, -0.884)	(0.996, -3.330)	(-1.413, 3.272)	(-3.777, 2.884)	(-2.807, 4.422)
(0.110, -0.845)	(1.211, 0.641)	(-0.200, -1.055)	(1.480, 1.487)	(-2.119, 6.399)	(-14.580, -15.487)
(0.110, -0.845)	(1.211, 0.641)	(2.963, -9.668)	(-8.840, 1.804)	(-2.119, 6.399)	(-14.580, -15.487)
(0.110, -0.845)	(1.211, 0.641)	(1.333, 0.679)	(6.033, 4.897)	(-1.151, 3.015)	(14.443, 7.940)
(0.110, -0.845)	(1.211, 0.641)	(0.650, -1.466)	(2.129, -0.416)	(-1.151, 3.015)	(14.443, 7.940)
(-1.462, 11.743)	(3.472, -0.884)	(3.395, 11.055)	(-6.601, -13.994)	(-1.151, 3.015)	(14.443, 7.940)
(-1.462, 11.743)	(3.472, -0.884)	(63.612, -75.345)	(2.610, 4.735)	(-1.151, 3.015)	(14.443, 7.940)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(3.6250, -2.9901) G(X,Y)=(0.8840, 1.5014)
H(X,Y)=(-0.9208, 2.8149) Q(X,Y)=(0.0164, 2.3893)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.269, -0.832)	(2.756, -0.164)	(-0.276, -0.829)	(0.798, 0.103)	(5.251, -2.468)	(0.819, 0.262)
(0.269, -0.832)	(2.756, -0.164)	(0.806, 1.112)	(3.677, -2.973)	(5.251, -2.468)	(0.819, 0.262)
(5.526, -4.928)	(5.694, -5.747)	(6.670, -6.293)	(1.329, -0.824)	(5.251, -2.468)	(0.819, 0.262)
(5.526, -4.928)	(5.694, -5.747)	(5.740, -2.187)	(-0.478, 2.691)	(5.251, -2.468)	(0.819, 0.262)
(-1.623, 5.545)	(-1.079, 4.958)	(-4.383, 4.889)	(-2.791, 0.423)	(5.251, -2.468)	(0.819, 0.262)
(-1.623, 5.545)	(-1.079, 4.958)	(-4.282, -1.743)	(-3.540, -7.171)	(5.251, -2.468)	(0.819, 0.262)
(-65.064, -43.527)	(19.230, 9.800)	(-0.032, -0.717)	(-0.574, -1.001)	(5.251, -2.468)	(0.819, 0.262)
(0.269, -0.832)	(2.756, -0.164)	(1.117, 0.419)	(1.718, 3.454)	(0.281, 2.566)	(3.687, 8.217)
(0.269, -0.832)	(2.756, -0.164)	(-1.594, -1.606)	(-0.177, 2.367)	(0.281, 2.566)	(3.687, 8.217)
(5.526, -4.928)	(5.694, -5.747)	(5.383, -4.902)	(0.419, 2.385)	(0.281, 2.566)	(3.687, 8.217)
(5.526, -4.928)	(5.694, -5.747)	(5.537, -5.451)	(1.618, 4.519)	(0.281, 2.566)	(3.687, 8.217)
(5.526, -4.928)	(5.694, -5.747)	(-1.592, -4.725)	(-3.679, -10.217)	(0.281, 2.566)	(3.687, 8.217)
(5.526, -4.928)	(5.694, -5.747)	(-12.519, -9.770)	(0.730, -11.842)	(0.281, 2.566)	(3.687, 8.217)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.480, 5.628)	(0.597, 2.485)	(0.281, 2.566)	(3.687, 8.217)
(-1.623, 5.545)	(-1.079, 4.958)	(-2.461, 5.381)	(0.236, 4.317)	(0.281, 2.566)	(3.687, 8.217)

TABLE XIV (continued)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(3.6250, -2.9901) G(X,Y)=(-1.1424, 6.0538)
H(X,Y)=(-0.9208, 2.8149) Q(X,Y)=(0.0163, 5.6100)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.269, -0.832)	(2.756, -0.164)	(1.074, -0.455)	(1.236, 1.660)	(0.174, 6.285)	(-0.727, -6.577)
(0.269, -0.832)	(2.756, -0.164)	(19.681, 5.087)	(-12.687, -11.038)	(0.174, 6.285)	(-0.727, -6.577)
(5.526, -4.928)	(5.694, -5.747)	(4.732, -5.061)	(3.038, -5.808)	(0.174, 6.285)	(-0.727, -6.577)
(5.526, -4.928)	(5.694, -5.747)	(5.353, -5.536)	(2.586, -7.723)	(0.174, 6.285)	(-0.727, -6.577)
(5.526, -4.928)	(5.694, -5.747)	(5.784, -5.028)	(4.894, 8.429)	(0.174, 6.285)	(-0.727, -6.577)
(5.526, -4.928)	(5.694, -5.747)	(-0.011, -2.141)	(4.296, -5.797)	(0.174, 6.285)	(-0.727, -6.577)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.931, 5.333)	(0.882, 5.533)	(0.174, 6.285)	(-0.727, -6.577)
(-1.623, 5.545)	(-1.079, 4.958)	(-3.026, 7.801)	(-1.504, 6.060)	(0.174, 6.285)	(-0.727, -6.577)
(0.269, -0.832)	(2.756, -0.164)	(0.242, -1.820)	(-0.047, 5.470)	(-0.144, 5.697)	(-2.221, -23.111)
(0.269, -0.832)	(2.756, -0.164)	(2.592, 2.401)	(7.318, 32.485)	(-0.144, 5.697)	(-2.221, -23.111)
(5.526, -4.928)	(5.694, -5.747)	(5.509, -5.038)	(0.488, 5.011)	(-0.144, 5.697)	(-2.221, -23.111)
(5.526, -4.928)	(5.694, -5.747)	(5.504, -5.635)	(2.882, 132.006)	(-0.144, 5.697)	(-2.221, -23.111)
(5.526, -4.928)	(5.694, -5.747)	(2.965, -5.402)	(-0.086, -8.821)	(-0.144, 5.697)	(-2.221, -23.111)
(5.526, -4.928)	(5.694, -5.747)	(33.778, -0.220)	(3.719, -8.225)	(-0.144, 5.697)	(-2.221, -23.111)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.589, 5.483)	(-0.257, 6.133)	(-0.144, 5.697)	(-2.221, -23.111)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.352, 5.021)	(-1.364, 5.899)	(-0.144, 5.697)	(-2.221, -23.111)
(0.269, -0.832)	(2.756, -0.164)	(-0.256, 0.207)	(1.259, 1.452)	(2.207, 5.849)	(1.558, 1.915)
(0.269, -0.832)	(2.756, -0.164)	(2.375, 2.653)	(3.169, 4.154)	(2.207, 5.849)	(1.558, 1.915)
(5.526, -4.928)	(5.694, -5.747)	(5.729, -5.262)	(1.665, 2.827)	(2.207, 5.849)	(1.558, 1.915)
(5.526, -4.928)	(5.694, -5.747)	(6.215, -5.995)	(-3.887, 3.482)	(2.207, 5.849)	(1.558, 1.915)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.367, 5.476)	(1.876, 4.750)	(2.207, 5.849)	(1.558, 1.915)
(-1.623, 5.545)	(-1.079, 4.958)	(-3.098, 7.169)	(-2.359, 3.876)	(2.207, 5.849)	(1.558, 1.915)
(-65.064, -43.527)	(19.230, 9.800)	(3.149, 0.443)	(4.511, 2.496)	(2.207, 5.849)	(1.558, 1.915)
(-65.064, -43.527)	(19.230, 9.800)	(-3.472, 0.240)	(-2.245, 2.355)	(2.207, 5.849)	(1.558, 1.915)
(-65.064, -43.527)	(19.230, 9.800)	(-11.825, 5.022)	(-10.835, 1.950)	(2.207, 5.849)	(1.558, 1.915)
(-65.064, -43.527)	(19.230, 9.800)	(-18.765, -570.846)	(-10.268, 27.797)	(2.207, 5.849)	(1.558, 1.915)
(0.269, -0.832)	(2.756, -0.164)	(2.649, 0.141)	(4.017, 1.797)	(5.535, 4.540)	(4.263, 2.007)
(0.269, -0.832)	(2.756, -0.164)	(4.346, 2.227)	(3.314, 0.591)	(5.535, 4.540)	(4.263, 2.007)
(5.526, -4.928)	(5.694, -5.747)	(3.109, -6.6[illegible]8)	(6.429, 1.378)	(5.535, 4.540)	(4.263, 2.007)
(5.526, -4.928)	(5.694, -5.747)	(7.189, -9.347)	(3.178, -1.530)	(5.535, 4.540)	(4.263, 2.007)
(-1.623, 5.545)	(-1.079, 4.958)	(1.404, 5.268)	(5.367, 1.365)	(5.535, 4.540)	(4.263, 2.007)
(-65.064, -43.527)	(19.230, 9.800)	(2.197, 0.464)	(3.038, 2.169)	(5.535, 4.540)	(4.263, 2.007)
(-65.064, -43.527)	(19.230, 9.800)	(157.437, 57.953)	(2.443, 0.430)	(5.535, 4.540)	(4.263, 2.007)

TABLE XIV (continued)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(3.6250, -2.9901) G(X,Y)=(0.2466, 3.9446)
H(X,Y)=(-0.9208, 2.8149) Q(X,Y)=(-0.0979, 3.9995)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.269, -0.832)	(2.756, -0.164)	(-0.698, -2.716)	(-0.480, 3.855)	(-0.063, 4.051)	(11.203, 59.590)
(0.269, -0.832)	(2.756, -0.164)	(1.574, 1.290)	(2.397, 10.105)	(-0.063, 4.051)	(11.203, 59.590)
(5.526, -4.928)	(5.694, -5.747)	(5.451, -4.961)	(0.000, 3.857)	(-0.063, 4.051)	(11.203, 59.590)
(5.526, -4.928)	(5.694, -5.747)	(5.551, -5.556)	(1.382, 12.327)	(-0.063, 4.051)	(11.203, 59.590)
(5.526, -4.928)	(5.694, -5.747)	(1.351, -5.216)	(-1.970, -9.693)	(-0.063, 4.051)	(11.203, 59.590)
(5.526, -4.928)	(5.694, -5.747)	(-80.547, -30.263)	(3.003, -9.841)	(-0.063, 4.051)	(11.203, 59.590)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.508, 5.576)	(0.176, 3.944)	(-0.063, 4.051)	(11.203, 59.590)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.432, 5.521)	(0.021, 4.695)	(-0.063, 4.051)	(11.203, 59.590)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.897, 5.372)	(-0.727, 5.268)	(-0.063, 4.051)	(11.203, 59.590)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.380, 5.514)	(0.023, 4.881)	(-0.063, 4.051)	(11.203, 59.590)
(0.269, -0.832)	(2.756, -0.164)	(1.406, -3.125)	(1.577, -2.822)	(0.027, 3.754)	(1.748, -5.294)
(0.269, -0.832)	(2.756, -0.164)	(1.826, -2.498)	(1.675, -2.178)	(0.027, 3.754)	(1.748, -5.294)
(0.269, -0.832)	(2.756, -0.164)	(1.506, -0.684)	(1.234, -0.000)	(0.027, 3.754)	(1.748, -5.294)
(0.269, -0.832)	(2.756, -0.164)	(-3.907, -4.678)	(-0.683, -3.544)	(0.027, 3.754)	(1.748, -5.294)
(5.526, -4.928)	(5.694, -5.747)	(5.740, -5.267)	(3.561, -0.639)	(0.027, 3.754)	(1.748, -5.294)
(5.526, -4.928)	(5.694, -5.747)	(-6.666, -3.686)	(5.031, -6.080)	(0.027, 3.754)	(1.748, -5.294)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.621, 5.494)	(-1.348, 5.499)	(0.027, 3.754)	(1.748, -5.294)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.659, 5.169)	(0.220, 4.074)	(0.027, 3.754)	(1.748, -5.294)
(0.269, -0.832)	(2.756, -0.164)	(5.747, -4.836)	(3.774, 1.924)	(1.269, 3.869)	(3.714, 1.118)
(0.269, -0.832)	(2.756, -0.164)	(2.826, 2.658)	(2.754, 3.833)	(1.269, 3.869)	(3.714, 1.118)
(5.526, -4.928)	(5.694, -5.747)	(3.662, -9.636)	(3.720, -3.473)	(1.269, 3.869)	(3.714, 1.118)
(5.526, -4.928)	(5.694, -5.747)	(7.245, -8.755)	(11.562, -5.335)	(1.269, 3.869)	(3.714, 1.118)
(-1.623, 5.545)	(-1.079, 4.958)	(-0.343, 4.914)	(7.546, -3.559)	(1.269, 3.869)	(3.714, 1.118)
(-1.623, 5.545)	(-1.079, 4.958)	(7.397, -5.477)	(1.332, 6.623)	(1.269, 3.869)	(3.714, 1.118)
(-65.064, -43.527)	(19.230, 9.800)	(3.789, 2.883)	(3.660, 4.208)	(1.269, 3.869)	(3.714, 1.118)
(0.269, -0.832)	(2.756, -0.164)	(0.132, -0.132)	(1.359, 1.924)	(-1.795, 3.664)	(1.323, 2.046)
(0.269, -0.832)	(2.756, -0.164)	(1.287, 2.598)	(0.074, 3.559)	(-1.795, 3.664)	(1.323, 2.046)
(5.526, -4.928)	(5.694, -5.747)	(5.869, -5.463)	(0.972, 2.110)	(-1.795, 3.664)	(1.323, 2.046)
(5.526, -4.928)	(5.694, -5.747)	(6.492, -6.260)	(0.844, 1.020)	(-1.795, 3.664)	(1.323, 2.046)
(-1.623, 5.545)	(-1.079, 4.958)	(-2.953, 5.085)	(-0.592, 2.639)	(-1.795, 3.664)	(1.323, 2.046)
(-1.623, 5.545)	(-1.079, 4.958)	(-2.731, 7.065)	(2.730, 5.245)	(-1.795, 3.664)	(1.323, 2.046)
(-65.064, -43.527)	(19.230, 9.800)	(1.050, 0.937)	(1.633, 3.352)	(-1.795, 3.664)	(1.323, 2.046)
(-65.064, -43.527)	(19.230, 9.800)	(-13.727, -167.204)	(-1.249, 2.070)	(-1.795, 3.664)	(1.323, 2.046)

TABLE XIV (continued)

THE FOLLOWING ARE THE SOLUTIONS CORRESPONDING TO
F(X,Y)=(3.6250, -2.9901) G(X,Y)=(31.9396, 18.0560)
H(X,Y)=(-0.9208, 2.8149) Q(X,Y)=(-9.7359, 4.0002)

M(X,Y)	A(X,Y)	B(X,Y)	C(X,Y)	D(X,Y)	E(X,Y)
(0.269, -0.832)	(2.756, -0.164)	(-1.821, 1.705)	(-4.589, 7.364)	(-3.777, 2.884)	(-2.807, 4.422)
(0.269, -0.832)	(2.756, -0.164)	(-3.689, 4.691)	(-4.454, 2.371)	(-3.777, 2.884)	(-2.807, 4.422)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.132, 5.678)	(-4.693, 1.307)	(-3.777, 2.884)	(-2.807, 4.422)
(-1.623, 5.545)	(-1.079, 4.958)	(-3.825, 7.941)	(-1.515, 2.729)	(-3.777, 2.884)	(-2.807, 4.422)
(-65.064, -43.527)	(19.230, 9.800)	(-60.423, -22.642)	(-5.436, 2.679)	(-3.777, 2.884)	(-2.807, 4.422)
(-65.064, -43.527)	(19.230, 9.800)	(-5.066, -4.733)	(-1.299, 1.128)	(-3.777, 2.884)	(-2.807, 4.422)
(0.269, -0.832)	(2.756, -0.164)	(-0.623, -1.083)	(1.370, 2.029)	(-2.119, 6.399)	(-14.580, -15.487)
(0.269, -0.832)	(2.756, -0.164)	(4.429, -1.643)	(11.387, 3.642)	(-2.119, 6.399)	(-14.580, -15.487)
(5.526, -4.928)	(5.694, -5.747)	(5.712, -4.702)	(9.210, 19.173)	(-2.119, 6.399)	(-14.580, -15.487)
(5.526, -4.928)	(5.694, -5.747)	(5.619, -5.385)	(-0.969, -12.092)	(-2.119, 6.399)	(-14.580, -15.487)
(5.526, -4.928)	(5.694, -5.747)	(3.554, -3.573)	(1.918, -4.734)	(-2.119, 6.399)	(-14.580, -15.487)
(5.526, -4.928)	(5.694, -5.747)	(-0.500, -0.383)	(2.616, -5.520)	(-2.119, 6.399)	(-14.580, -15.487)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.402, 5.614)	(0.905, 7.067)	(-2.119, 6.399)	(-14.580, -15.487)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.178, 5.880)	(0.795, 7.496)	(-2.119, 6.399)	(-14.580, -15.487)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.944, 5.709)	(1.543, 5.890)	(-2.119, 6.399)	(-14.580, -15.487)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.706, 2.735)	(-0.209, 6.943)	(-2.119, 6.399)	(-14.580, -15.487)
(-65.064, -43.527)	(19.230, 9.800)	(-48.781, -34.524)	(-1.185, 1.384)	(-2.119, 6.399)	(-14.580, -15.487)
(-1.623, 5.545)	(-1.079, 4.958)	(-1.296, 5.775)	(2.855, -0.013)	(-1.151, 3.015)	(14.443, 7.940)
(-1.623, 5.545)	(-1.079, 4.958)	(-3.793, 4.902)	(1.015, 4.543)	(-1.151, 3.015)	(14.443, 7.940)
(-65.064, -43.527)	(19.230, 9.800)	(-0.072, -6.919)	(5.448, -0.009)	(-1.151, 3.015)	(14.443, 7.940)
(-65.064, -43.527)	(19.230, 9.800)	(-105.901, -10.147)	(-16.768, 6.127)	(-1.151, 3.015)	(14.443, 7.940)

TOTAL NUMBER OF MECHANISMS= 156

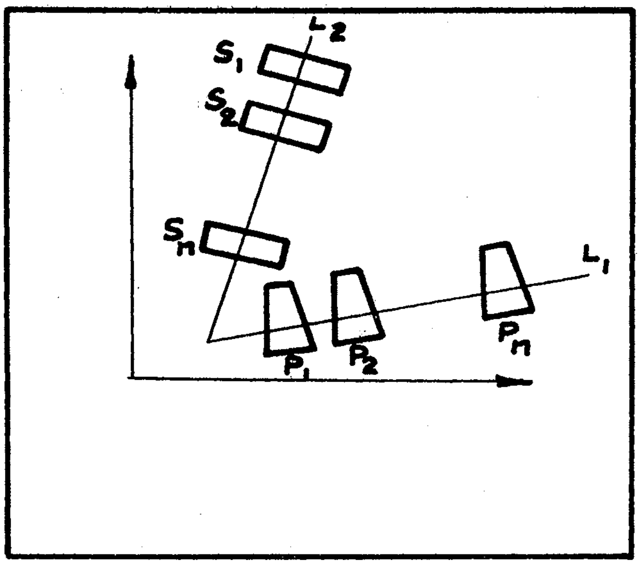


Figure 14. Non-Orthagonal Rectilinear
Paths of Rigid Bodies

translations. As pointed out by Hain [47], the mechanism of case 2 cannot be used for this purpose. However the eight-link mechanism of Figure 11 (Hain's eight-bar) can be used to generate two rectilinear translations (approximated by five precision positions).

When two rectilinear motions are prescribed then step (1) of case 1 of the preceding section fails to provide a solution. Hence the pivots C and D (Figure 11) will have to be specified in all positions. Thus the rectilinear translations are specified by specifying positions of C and D on two straight lines of arbitrary orientation in XY-plane. Obviously then the last 4 positions of D are fixed once the 5 positions of C, the first position of D and the slope of the axis of translation by D, are prescribed.

The synthesis procedure then is to start at equation (6-3) and follow through the procedure of the case 1.

Examples are given in Tables XV and XVI for general orientation of axes of translations, and for the case when these axes are at right angles to each other.

Numerical Solutions

The nature of the input specifications, to the problem, namely the motion of two rigid bodies, determines the number of real solutions that exist. The synthesis procedure presented in this paper has been found to be highly sensitive in terms of numerical computations. Hence a small change in input data may reflect in a significant change in the number and values of the solutions.

TABLE XV

NON-ORTHAGONAL RECTILINEAR MOTION GENERATION

DESIGN SPECIFICATIONS

5.50000	1.87542	3.00000	9.00000
7.00000	2.74145	2.93184	9.10224
8.25000	3.46314	3.04487	8.93270
9.25000	4.04049	3.25032	8.62452
10.00000	4.47350	3.47960	8.28059

A	B	C	D	MECHANISM PARAMETERS					
1				A(	4.4795970,	5.5232760)	M(	7.1203610,	2.8278800)
	1			B(	4.4797220,	5.5232350)	E(	5.5003370,	1.8750780)
		1		F(	21.9265800,	29.1462200)	H(	2.9999850,	8.9999470)
			1	G(	4.4858550,	5.4536890)	Q(	7.2309370,	2.5838520)
			2	G(	4.5554190,	9.3006890)	Q(	1206.5790000,	485.7570000)
			3	G(	4.7643280,	1.7956690)	Q(	457.3010000,	-991.3920000)
			4	G(	8.2116770,	6.1362910)	Q(	17358.4600000,	-5671.5700000)
		2		F(	2.9996660,	9.0001540)	H(	-23.6311100,	10.6214700)
				NO SOLUTION EXISTS FOR THIS STEP					
	2			B(	7.3905640,	3.1676790)	E(	942.5063000,	-1423.9800000)
				NO SOLUTION EXISTS FOR THIS STEP					

TOTAL NUMBER OF MECHANISMS= 4

TABLE XVI

ORTHAGONAL RECTILINEAR MOTION GENERATION

DESIGN SPECIFICATIONS

7.12000	2.61600	3.00000	8.00000
8.10000	3.15500	3.12920	7.76509
8.80000	3.54000	3.28466	7.48244
9.00000	3.65000	3.34000	7.38181
9.10000	3.70500	3.36967	7.32787

A	B	C	D	MECHANISM PARAMETERS					
1				A(	5.0248365,	4.7095957)	M(	6.6531744,	-0.7282295)
	1			B(	4.9455376,	4.7458305)	E(	6.9127111,	2.7505140)
		1		F(	2.4233513,	8.7050285)	H(	2.8317394,	8.1005707)
			1	G(	3.8780479,	5.2769423)	Q(	5.2961884,	-7.0173798)
			2	G(	4.6314974,	8.5260105)	Q(	-18.6295471,	7.4687958)
			3	G(	4.0881462,	6.1378508)	Q(	4.8968658,	2.8755960)
			4	G(	2.1588707,	7.1032982)	Q(	-1225.5361328,	-1239.6735840)
		2		F(	3.1103745,	7.8229561)	H(	5.2408028,	6.3123198)
			1	G(	-18.1669159,	-7.0411644)	Q(	-94.7713013,	-51.9801178)

TABLE XVI (continued)

	2			B(17.8779144, -1.2271080)	E(-18.1394958, 19.0858917)
		1		F(-18.7903137, 19.5992737)	H(-23.3330383, 25.4251556)
			1	G(-21.1170197, 21.9175415)	Q(-19.3949738, 16.4891052)
			2	G(-21.2689819, 15.9870214)	Q(-25.4951324, 53.1376038)
			3	G(-20.9946136, 26.1756897)	Q(23.6680756, 33.7428589)
			4	G(-24.7045441, 23.3363342)	Q(*************,*************)
		2		F(-0.7833710, 6.0381937)	H(18.9919586, 28.8507385)
			1	G(-5.0429974, 28.2994232)	Q(-3.3313627, 28.6808777)
		3		F(98.0846405, -69.2855377)	H(39.1222992, -28.6595612)
				NO SOLUTION EXISTS FOR THIS STEP	
	3			B(1.2032118, 6.4871778)	E(-17.2281342, -21.5760956)
		1		F(-14.5098705, -18.6635590)	H(-19.1263580, -12.8500671)
			1	G(-16.7168427, -15.2465487)	Q(-15.7811289, -17.1509857)
			2	G(-16.8232574, -23.8723602)	Q(-21.0498047, 12.8332815)
			3	G(-16.8984528, -12.0523682)	Q(-99.7919312, -29.2447968)
			4	G(-15.1742201, -12.3541565)	Q(572.0732422, -20.4523926)
		2		F(26.8932648, 28.3477936)	H(38.5187378, 20.1443024)
			1	G(35.9123840, 15.8945370)	Q(35.8176422, 15.7073259)
			2	G(18.8033752, 34.1185455)	Q(26.6463776, 27.5249786)
		3		F(12.8277435, 10.3883972)	H(-154.7090302, 156.8157959)
			1	G(13.6295271, -19.2047577)	Q(18.4477081, -25.8884735)
			2	G(-78.2328033, 198.1645355)	Q(-206.5969696, 129.7857666)
		4		F(15.4934692, 9.3070984)	H(-509.7094727, 464.4287109)
			1	G(7.3531170, -93.9589996)	Q(11.9751377, -100.5644073)
			2	G(-44.2729187, 141.0254517)	Q(-41.5277252, 138.3122711)
	4			B(4.4821119, 9.2625742)	E(*************,-6525.6718750)
				NO SOLUTION EXISTS FOR THIS STEP	
2				A(5.2629747, 8.6912489)	M(-15.8625507, 5.6601105)
	1			B(5.4048338, 8.7681761)	E(7.2373228, 2.7375822)
		1		F(3.1495581, 8.0176125)	H(3.1530294, 8.1459026)
			1	G(7.0802708, 12.3775911)	Q(6.8425140, 12.0862885)
			2	G(7.3920298, 9.8641024)	Q(8.2261200, 10.1557617)
			3	G(7.6212358, 8.1060991)	Q(7.8495941, 8.0440674)
			4	G(2.9823599, 8.0281086)	Q(568.1801758, 325.3496094)
		2		F(3.1559677, 8.0839033)	H(3.6474543, 8.7475700)
			1	G(43.6849518, 157.2257385)	Q(43.3576050, 156.7287903)
			2	G(156.6084900, -42.6447601)	Q(157.1670227, -42.6547089)
			3	G(-18.1445313, -3.2673693)	Q(4264.9726563, 2352.1018066)
	2			B(-8.8748236, 1.0121088)	E(5.4865942, -27.7998810)
		1		F(5.7475061, -28.0591278)	H(1.5416508, -22.5783997)
			1	G(3.7089415, -23.9734955)	Q(4.2569151, -24.7557831)
			2	G(3.8209581, -21.8784943)	Q(-3.1527939, -22.8465576)
			3	G(3.5489330, -27.0840759)	Q(7.0317030, -42.8077087)

TABLE XVI (continued)

			4	G(0.2711420, -26.0832977)	Q(4346.6679688, 7991.6445313)
		2		F(-37.1966553, 12.3009338)	H(-41.5668640, 14.8132629)
			1	G(-44.3781891, 13.6999350)	Q(-44.2296143, 13.8229389)
			2	G(-39.4190063, 10.7506399)	Q(-34.0632324, 4.6151094)
			3	G(-43.5073242, 13.1608725)	Q(-59.9738312, -5.4430494)
			4	G(-43.2411346, 13.5637512)	Q(405.8435059, 462.5068359)
		3		F(-19.4560699, -4.1752777)	H(-71.6337128, -64.8084717)
			1	G(-8.8502016, 19.4733429)	Q(-10.1776733, 18.2213593)
			2	G(76.3538208, -83.2805634)	Q(77.3866730, -83.2535553)
		4		F(-13.1584253, 3.6244354)	H(-3401.4035645,-4542.1875000)
				NO SOLUTION EXISTS FOR THIS STEP	
	3			B(-38.5817413, -14.9216204)	E(-41.7082825, -11.9360781)
		1		F(-37.4479218, -13.7069407)	H(-5.1921234, 23.5565643)
			1	G(-16.4289246, 5.1072226)	Q(-17.4900970, 3.9499311)
			2	G(-46.0659637, 1.3251791)	Q(9899.5117188, 5464.3750000)
		2		F(-40.8550720, -12.2910604)	H(-42.5473938, -9.3559723)
			1	G(-41.7284698, -11.8782797)	Q(-110.3199310, 635.3618164)
		3		F(-43.1706390, -11.3471832)	H(-60.6361389, 5.9775915)
			1	G(-51.0357056, -3.2878466)	Q(-48.4966583, -7.0599203)
			2	G(-69.6023407, 62.8471375)	Q(-63.4247742, 32.8132629)
			3	G(-58.8366547, 7.0053253)	Q(-1396.2216797, -654.2307129)
3				A(4.8089981, 1.8806791)	M(-131.5706329, 982.1264648)
	1			B(2.0167685, 0.2926464)	E(4.3681507, 1.0456810)
		1		F(0.2124643, 6.5022058)	H(0.0814085, 5.4481468)
			1	G(45.8052673, 30.5799561)	Q(16.6187897, 14.9304047)
			2	G(10.1879282, 11.0766535)	Q(-723.1223145, -390.6437988)
	2			B(8.1874905, 3.7719173)	E(10.4499969, 4.4862051)
				NO SOLUTION EXISTS FOR THIS STEP	
4				A(4.9615355, 1.9402952)	M(360.6582031,-2171.4074707)
	1			B(7.5752068, 2.6127977)	E(9.7490883, 3.2928286)
				NO SOLUTION EXISTS FOR THIS STEP	
	2			B(3.7517309, -12.5865917)	E(5.8789616, -11.9640112)
		1		F(1.8935719, -6.5546932)	H(9.3391886, 2.2909451)
				NO SOLUTION EXISTS FOR THIS STEP	
		2		F(4.4137297, -11.1295366)	H(1.7742739, -6.5848598)
			1	G(3.8607893, -9.0111704)	Q(4.8823318, -11.4974422)
			2	G(3.8121834, -11.2779579)	Q(8.3312140, -31.7585449)
			3	G(3.9958172, -5.8913279)	Q(-17.3203888, -9.0722713)
			4	G(0.4384584, -9.3125200)	Q(880.5239258, 1262.3259277)

TOTAL NUMBER OF MECHANISMS= 49

Interpretation of the Tables of Numerical Solution

Table XIII shows the design specifications and the 93 mechanisms obtained. The letters A, B, C, D and E correspond to the step a, b, c, d and e of the synthesis procedure illustrated in Figure 12. The number under these letters identify the particular mechanism parameters obtained for that step. For each step there may exist as many as four solutions. Hence a complete mechanism may be obtained be combining together the design parameters one from each, associated, step. Symbolically for Table XIII,

$$\text{Mechanism } (A,B,C,D,E) = \begin{matrix} C(x, y) & D(x, y) \\ A(x, y) & M(x, y) \\ B(x, y) & E(x, y) \\ F(x, y) & H(x, y) \\ G(x, y) & Q(x, y) \end{matrix}$$

For example

$$\text{Mechanism } (1,1,1,2,2) = \begin{matrix} C(1.2761459, 1.2192926) \\ A(1.2113590, 0.6412295) \\ B(1.2113876, 0.6412346) \\ F(2.4600410, 1.7371149) \\ G(3.4753065, -0.8893805) \\ \\ D \\ M \\ E \\ H \\ Q \end{matrix}$$

Obviously the total number of solutions is the sum of the number of entries in column E.

Table XIV is self-explanatory.

Table XV shows the 4 mechanisms obtained for non-orthogonal rectilinear translations problem. In the input data the exact expression for FY is,

$$CY = \tan(30^{\circ})\ CX - 1.3$$

and the slope of the line of translation of the point D is -1.5.

Table XVI shows the 49 mechanisms obtained for orthogonal rectilinear translations problem. The numerical results in Tables XV and XVI are presented in the same manner as in Table XIII. In the two problems of rectilinear translations the 2nd through 5th positions of the pivot D are computed from the first position of D, the five positions of the pivot C and the slope of translation of D (which is -1.5 in the example of Table XV).

CHAPTER VII

SUMMARY AND CONCLUSIONS

This work provides a generalized approach to the dimensional synthesis of multi-loop planar mechanisms. The synthesis technique is illustrated by considering two eight-link mechanisms for the following types of synthesis problems:

(1) Coupler Point-Path Generation,

(2) Coupler Point-Path Generation coordinated with the Angular Displacements of Input Link,

(3) Coupler Point-Path Generation coordinated with the Angular Displacements of Input and Output Links,

(4) Rigid Body Guidance,

(5) Rigid Body Guidance coordinated with the Angular Displacements of the Input Link,

(6) Rigid Body Guidance coordinated with the Angular Displacements of the Input and Output Links,

(7) Coordination of the Angular Displacements of Input and Output Links,

(8) Generation of Two Coupler Point-Paths,

(9) Simultaneous Non-Rectilinear Motion Generation of two Rigid Bodies:

Case 1. Synthesis of Hain's Eight-Link Mechanism,

Case 2. Synthesis of the Eight-Link Mechanism having Five Links in each of its loops,

(10) Simultaneous Rectilinear Motion Generation of two Rigid Bodies.

The other 69, out of 71, eight-link mechanisms can be synthesized using the proposed generalized approach.

The general synthesis procedure has three steps:

(1) Identification of the angles that should be treated as unknown parameters in the synthesis equations.

(2) Derivation of the system of design equations by imposing kinematic constraints to insure one degree of freedom mechanism.

(3) Numerical solution of the system of design equations.

The design equations obtained in step (2) are highly nonlinear and transcendental in nature. The number of these equations depends upon the functional specifications. For example, for the synthesis problem of path generation, for 21 precision positions, there are 100 synthesis equations. Similarly for other problems different number of equations are obtained. Table IV shows, for the first eight design problems, the relationship between the number of precision positions and various other parameters like the number of design equations, number of unknown variables, etc.

In the case of multiple rigid bodies guidance problem it is possible to linearize the system of nonlinear equations by invoking the principle of linear superposition. In this case, the resulting equations can be solved in a closed form to obtain all theoretically possible solutions. This type of linearization is not possible in other cases if more than five precision conditions are specified.

In those cases the equations are to be solved numerically on a computer. For this purpose the problem of solving nonlinear algebraic equations has been converted into an optimization problem which has been solved by using Marquardt numerical technique. This numerical technique has been found capable of providing good convergence with reasonably good initial guesses for the unknown parameters.

The present work

(a) provides a general approach to synthesize planar mechanisms with revolute pairs,

(b) contributes mathematical approach to calculate the maximum number of precision conditions than can be specified for a variety of functional specifications,

(c) reveals the practicability of Marquardt's numerical technique to solve synthesis equations,

(d) provides a "segmented approach" to synthesize eight link mechanisms for multiple rigid bodies guidance by splitting the synthesis problem into a number of identical and simple problems.

In addition, the proposed linearized approach yields all possible solutions which are capable of performing the same job. From a practical standpoint the designer can have his own choice on the basis of some desirable characteristics like transmission, space requirements, etc.

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APPENDIX

COMPUTER PROGRAMS

80/80 LIST

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12345678901234567890123456789012345678901234567890123456789012345678901234567890

C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       *                                                                       *
C       *                                                                       *
C       *    A GENERAL DESCRIPTION OF PARAMETERS USED IN PROGRAMS OTHER         *
C       *                                                                       *
C       *    THAN THE TWO RIGID BODIES GUIDANCE PROGRAMS                        *
C       *        NE,KZ - NO. OF EQUATIONS/VARIBLES                              *
C       *        NITER - MAX. NO. OF ITERATIONS PERMITTED                       *
C       *        NP - NO. OF PRECISION POINTS                                   *
C       *        NN - NO. OF ANGULAR DISPLACEMENTS (NN=NP-1)                    *
C       *        SPAR - DEFINED IN SUBROUTINE SOLVE(MRQT)                       *
C       *        XP,YP - COORDINATES OF COUPLER POINT 'P'                       *
C       *        XS,YS - COORDINATES OF COUPLER POINT 'S'                       *
C       *        ICON - ERROR MESSAGE DESCRIBED IN SUBROUTINE SOLVE(MRQT)       *
C       *        Z - ARRAY OF UNKNOWNS                                          *
C       *        PF - ARRAY OF RESIDUALS                                        *
C       *                                                                       *
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       *                                                                     *
C       *                                                                     *
C       *    PURPOSE                                                          *
C       *        TO DESIGN EIGHT LINK MECHANISM FOR POINT PATH GENERATION     *
C       *                                                                     *
C       *    INPUT  DATA                                                      *
C       *        CARD 1. KZ,NE,NITER,NN,NP WITH 5I3                           *
C       *        CARD 2. SPAR WITH 4F10.0                                     *
C       *        CARD 3. Z WITH 8F10.0                                        *
C       *        CARD 4. XP,YP WITH 8F10.0                                    *
C       *                                                                     *
C       *    SUBROUTINES REQUIRED                                             *
C       *        SUBROUTINE SOLVE(MRQT)                                       *
C       *                                                                     *
C       *                                                                     *
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DOUBLE PPECISION P ,ZMIN,QI,Z,DE,ZMAX
      COMMON XP(21),YP(21),NN
      DIMENSION Z(100),Y(100),PF(100)
      DIMENSION QI(100,100),DE(100)
      DIMENSION ZMIN(100),ZMAX(100),SPAR(4)
    1 FORMAT(5I3)
    2 FORMAT(4F10.7)
    3 FORMAT(8F10.0)
    4 FORMAT(10X,'ICON=',I3,5X,'NO. OF ITR =',I3,5X,'RESIDUAL=',D13.6)
    5 FORMAT(10X,' STARTING MECHANISM ....',/,8(F13.5,3X))
    6 FORMAT(10X,' PRECISION POINTS ...',/,8(F13.8,3X))
    7 FORMAT(10X,'RESIDUALS ...',/,8(E13.5,3X))
    8 FORMAT(10X,' FINAL MECHANISM... ',/,8(F13.5,3X))
      KR=5
      KW=6
      DSIG=1.0
      READ(KR,1) KZ,NE,NITER,NN,NP
      DO 9 I=1,KZ
      ZMIN(I)=-360.
    9 ZMAX(I)= 360.
      READ(KR,2)( SPAR(J),J=1,4)
      DO 10 I=1,NE
   10 Y(I)=0.0
      READ(KR,3 )(Z(I),I=1,KZ)
      WRITE(KW,5)(Z(I),I=1,KZ)
      M=0
      READ(KR, 3) (XP(J),YP(J),J=1,NP)
      WRITE(KW,6) (XP(J),YP(J),J=1,NP)
   11 CALL SOLVE(KZ,NE,M,ICON,KERR,Z,ZMIN,ZMAX,Y,PF,P,SPAR,QI,DE,KW,
     *  NITER,DSIG)
      WRITE(KW,4)ICON,M,P
      WRITE(KW,8) (Z(J),J=1,KZ)
      WRITE(KW,7 )(PF(J),J=1,NE)
      IF(M.EQ.NITER) GO TO 12
      IF(P.GT..0001) GO TO 11
   12 S T O P
```

80/80 LIST

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      END                                                               PPG 0370
      SUBROUTINE DESIGN(KZ,Z,NE,EQ,P,NAZ)                               PPG 0380
      DOUBLE PRECISION Z                                                PPG 0390
      COMMON XP(21),YP(21),NN                                           PPG 0400
      DIMENSION Z(KZ),EQ(NE),P(NE,KZ)                                   PPG 0410
      DIMENSION X(5),Y(5)                                               PPG 0420
      IF(NAZ.GT.0) GO TO 2                                              PPG 0430
      DO 1 I=1,NE                                                       PPG 0440
      DO 1 J=1,KZ                                                       PPG 0450
    1 P(I,J)=0.0                                                        PPG 0460
    2 GG=57.29578                                                       PPG 0470
      BC1=(Z(5)-Z(7))*(Z(5)-Z(7))+(Z(6)-Z(8))*(Z(6)-Z(8))               PPG 0480
      FE1=(Z(13)-Z(11))*(Z(13)-Z(11))+(Z(14)-Z(12))*(Z(14)-Z(12))       PPG 0490
      FP1=(Z(13)-XP(1))*(Z(13)-XP(1))+(Z(14)-YP(1))*(Z(14)-YP(1))       PPG 0500
      EP1=(Z(11)-XP(1))*(Z(11)-XP(1))+(Z(12)-YP(1))*(Z(12)-YP(1))       PPG 0510
      GH1=(Z(15)-Z(17))*(Z(15)-Z(17))+(Z(16)-Z(18))*(Z(16)-Z(18))       PPG 0520
      DO 5 I=1,NN                                                       PPG 0530
      J=I+1                                                             PPG 0540
      J1=NN+I                                                           PPG 0550
      J2=J1+NN                                                          PPG 0560
      J3=J2+NN                                                          PPG 0570
      J4=J3+NN                                                          PPG 0580
      T=Z(J4)/GG                                                        PPG 0590
      B=Z(J2)/GG                                                        PPG 0600
      F=Z(J3)/GG                                                        PPG 0610
      G=Z(J1)/GG                                                        PPG 0620
      DT11= COS(T)                                                      PPG 0630
      DB11= COS(B)                                                      PPG 0640
      DF11= COS(F)                                                      PPG 0650
      DG11= COS(G)                                                      PPG 0660
      DT12=-SIN(T)                                                      PPG 0670
      DB12=-SIN(B)                                                      PPG 0680
      DF12=-SIN(F)                                                      PPG 0690
      DG12=-SIN(G)                                                      PPG 0700
      VT=1.0-DT11                                                       PPG 0710
      VF=1.0-DF11                                                       PPG 0720
      DT13= Z(1)*VT-Z(2)*DT12                                           PPG 0730
      DF13= Z(19)*VF-Z(20)*DF12                                         PPG 0740
      DT23= Z(2)*VT+Z(1)*DT12                                           PPG 0750
      DF23= Z(20)*VF+Z(19)*DF12                                         PPG 0760
      AXN= DT11*Z(3)+DT12*Z(4)+DT13                                     PPG 0770
      AYN=-DT12*Z(3)+DT11*Z(4)+DT23                                     PPG 0780
      BXN= DT11*Z(5)+DT12*Z(6)+DT13                                     PPG 0790
      BYN=-DT12*Z(5)+DT11*Z(6)+DT23                                     PPG 0800
      DXN= DF11*Z(9)+DF12*Z(10)+DF13                                    PPG 0810
      DYN=-DF12*Z(9)+DF11*Z(10)+DF23                                    PPG 0820
      GXN= DF11*Z(15)+DF12*Z(16)+DF13                                   PPG 0830
      GYN=-DF12*Z(15)+DF11*Z(16)+DF23                                   PPG 0840
      DG13=AXN-DG11*Z(3)-DG12*Z(4)                                      PPG 0850
      DG23=AYN+DG12*Z(3)-DG11*Z(4)                                      PPG 0860
      DB13=DXN-DB11*Z(9)-DB12*Z(10)                                     PPG 0870
      DB23=DYN+DB12*Z(9)-DB11*Z(10)                                     PPG 0880
      FXN= DG11*Z(13)+DG12*Z(14)+DG13                                   PPG 0890
      FYN=-DG12*Z(13)+DG11*Z(14)+DG23                                   PPG 0900
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      HXN= DG11*Z(17)+DG12*Z(18)+DG13                                   PPG 0910
      HYN=-DG12*Z(17)+DG11*Z(18)+DG23                                   PPG 0920
      CXN= DB11*Z(7)+DB12*Z(8)+DB13                                     PPG 0930
      CYN=-DB12*Z(7)+DB11*Z(8)+DB23                                     PPG 0940
      EXN= DB11*Z(11)+DB12*Z(12)+DB13                                   PPG 0950
      EYN=-DB12*Z(11)+DB11*Z(12)+DB23                                   PPG 0960
      EQ(1)=(BXN-CXN)*(BXN-CXN)+(BYN-CYN)*(BYN-CYN)-BC1                 PPG 0970
      EQ(J1)=(FXN-EXN)*(FXN-EXN)+(FYN-EYN)*(FYN-EYN)-FE1                PPG 0980
      EQ(J2)=(FXN-XP(J))*(FXN-XP(J))+(FYN-YP(J))*(FYN-YP(J))-FP1        PPG 0990
      EQ(J3)=(EXN-XP(J))*(EXN-XP(J))+(EYN-YP(J))*(EYN-YP(J))-EP1        PPG 1000
      EQ(J4)=(GXN-HXN)*(GXN-HXN)+(GYN-HYN)*(GYN-HYN)-GH1                PPG 1010
      IF( NAZ .GT. 0) GO TO 5                                           PPG 1020
      X(1)=BXN-CXN                                                      PPG 1030
      Y(1)=BYN-CYN                                                      PPG 1040
      X(2)=FXN-EXN                                                      PPG 1050
      Y(2)=FYN-EYN                                                      PPG 1060
      X(3)=FXN-XP(J)                                                    PPG 1070
      Y(3)=FYN-YP(J)                                                    PPG 1080
      X(4)=EXN-XP(J)                                                    PPG 1090
      Y(4)=EYN-YP(J)                                                    PPG 1100
      X(5)=GXN-HXN                                                      PPG 1110
      Y(5)=GYN-HYN                                                      PPG 1120
      D1=DT11-DG11                                                      PPG 1130
      D2=-DT12+DG12                                                     PPG 1140
      D3= DF11-DB11                                                     PPG 1150
      D4=-DF12+DB12                                                     PPG 1160
      DO 3 II=1,3                                                       PPG 1170
      IN=1+NN*(II-1)                                                    PPG 1180
      P(IN,1)=X(II)*VT+Y(II)*DT12                                       PPG 1190
      P(IN,2)=-X(II)*DT12+Y(II)*VT                                      PPG 1200
      IA=II*(II-1)/2+2                                                  PPG 1210
      IO=I+NN*(IA-1)                                                    PPG 1220
      P(IO,3)= X(IA)*D1+Y(IA)*D2                                        PPG 1230
      P(IO,4)=-X(IA)*D2+Y(IA)*D1                                        PPG 1240
      IB=IA-1                                                           PPG 1250
      IP=1+NN*(IB-1)                                                    PPG 1260
      P(IP,9)=-X(IB)*D3-Y(IB)*D4                                        PPG 1270
    3 P(IP,10)= X(IB)*D4-Y(IB)*D3                                       PPG 1280
      P(J4,1)=-X(5)*VT-Y(5)*DT12                                        PPG 1290
      P(J4,2)= X(5)*DT12-Y(5)*VT                                        PPG 1300
      P(J4,3)=-P(J4,3)                                                  PPG 1310
      P(J4,4)=-P(J4,4)                                                  PPG 1320
      P(I,5)= X(1)*DT11-Y(1)*DT12-Z(5)+Z(7)                             PPG 1330
      P(I,6)= X(1)*DT12+Y(1)*DT11-Z(6)+Z(8)                             PPG 1340
      P(I,7)=-X(1)*DB11+Y(1)*DB12+Z(5)-Z(7)                             PPG 1350
      P(I,8)=-X(1)*DB12-Y(1)*DB11+Z(6)-Z(8)                             PPG 1360
      P(J3,9)=-P(J3,9)                                                  PPG 1370
      P(J3,10)=-P(J3,10)                                                PPG 1380
      P(J1,11)=-X(2)*DB11+Y(2)*DB12+Z(13)-Z(11)                         PPG 1390
      P(J3,11)= X(4)*DB11-Y(4)*DB12-Z(11)+XP(1)                         PPG 1400
      P(J1,12)=-X(2)*DB12-Y(2)*DB11+Z(14)-Z(12)                         PPG 1410
      P(J3,12)= X(4)*DB12+Y(4)*DB11-Z(12)+YP(1)                         PPG 1420
      P(J1,13)= X(2)*DG11-Y(2)*DG12-Z(13)+Z(11)                         PPG 1430
      P(J2,13)= X(3)*DG11-Y(3)*DG12-Z(13)+XP(1)                         PPG 1440
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      P(J1,14)= X(2)*DG12+Y(2)*DG11-Z(14)+Z(12)                         PPG 1450
      P(J2,14)= X(3)*DG12+Y(3)*DG11-Z(14)+YP(1)                         PPG 1460
      P(J4,15)= X(5)*DF11-Y(5)*DF12-Z(15)+Z(17)                         PPG 1470
      P(J4,16)= X(5)*DF12+Y(5)*DF11-Z(16)+Z(18)                         PPG 1480
      P(J4,17)=-X(5)*DG11+Y(5)*DG12+Z(15)-Z(17)                         PPG 1490
      P(J4,18)=-X(5)*DG12-Y(5)*DG11+Z(16)-Z(18)                         PPG 1500
      DO 4 K=1,2                                                        PPG 1510
      IQ=I+NN*(K-1)                                                     PPG 1520
      IR=I+NN*(K+2)                                                     PPG 1530
      IC=K+3                                                            PPG 1540
      P(IQ,19)=-X(K)*VF-Y(K)*DF12                                       PPG 1550
      P(IR,19)= X(IC)*VF+Y(IC)*DF12                                     PPG 1560
      P(IQ,20)= X(K)*DF12-Y(K)*VF                                       PPG 1570
    4 P(IR,20)=-X(IC)*DF12+Y(IC)*VF                                     PPG 1580
      D5=Z(13)*DG12-Z(14)*DG11-Z(3)*DG12+Z(4)*DG11                      PPG 1590
      D6=Z(13)*DG11+Z(14)*DG12-Z(3)*DG11-Z(4)*DG12                      PPG 1600
      P(J1,J1)= X(2)*D5+Y(2)*D6                                         PPG 1610
      P(J2,J1)= X(3)*D5+Y(3)*D6                                         PPG 1620
      D7=Z(17)*DG12-Z(18)*DG11-Z(3)*DG12+Z(4)*DG11                      PPG 1630
      D8=Z(17)*DG11+Z(18)*DG12-Z(3)*DG11-Z(4)*DG12                      PPG 1640
      P(J4,J1)=-X(5)*D7-Y(5)*D8                                         PPG 1650
      D9=-Z(9)*DB12+Z(10)*DB11                                          PPG 1660
      D10=-Z(9)*DB11-Z(10)*DB12                                         PPG 1670
      P(I,J2)=-X(1)*(Z(7)*DB12-Z(8)*DB11+D9)-Y(1)*(Z(7)*DB11            PPG 1680
     C +Z(8)*DB12+D10)                                                  PPG 1690
      D11=Z(11)*DB12-Z(12)*DB11+D9                                      PPG 1700
      D12=Z(11)*DB11+Z(12)*DB12+D10                                     PPG 1710
      P(J1,J2)=-X(2)*D11-Y(2)*D12                                       PPG 1720
      P(J3,J2)= X(4)*D11+Y(4)*D12                                       PPG 1730
      D13=(Z(9)-Z(19))*DF12-(Z(10)-Z(20))*DF11                          PPG 1740
      D14=(Z(9)-Z(19))*DF11+(Z(10)-Z(20))*DF12                          PPG 1750
      P(I,J3)=-X(1)*D13-Y(1)*D14                                        PPG 1760
      P(J1,J3)=-X(2)*D13-Y(2)*D14                                       PPG 1770
      P(J3,J3)= X(4)*D13+Y(4)*D14                                       PPG 1780
      D15=Z(15)-Z(19)                                                   PPG 1790
      D16=Z(16)-Z(20)                                                   PPG 1800
      P(J4,J3)= X(5)*(D15*DF12-D16*DF11)+Y(5)*(D15*DF11+D16*DF12)       PPG 1810
      D17=Z(5)-Z(1)                                                     PPG 1820
      D18=Z(6)-Z(2)                                                     PPG 1830
      P(I,J4)= X(1)*(D17*DT12-D18*DT11)+Y(1)*(D17*DT11+D18*DT12)        PPG 1840
      D19=Z(3)-Z(1)                                                     PPG 1850
      D20=Z(4)-Z(2)                                                     PPG 1860
      D21=D19*DT12-D20*DT11                                             PPG 1870
      D22=D19*DT11+D20*DT12                                             PPG 1880
      P(J1,J4)= X(2)*D21+Y(2)*D22                                       PPG 1890
      P(J2,J4)= X(3)*D21+Y(3)*D22                                       PPG 1900
      P(J4,J4)=-X(5)*D21-Y(5)*D22                                       PPG 1910
    5 C O N T I N U E                                                   PPG 1920
      IF( NAZ .GT. 0 ) RETURN                                           PPG 1930
      DO 6 I=1,NE                                                       PPG 1940
      DO 6 J=1,KZ                                                       PPG 1950
    6 P(I,J)=P(I,J)/2.0                                                 PPG 1960
      R E T U R N                                                       PPG 1970
      E N D                                                             PPG 1980
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      SUBROUTINE FUNC(KZ,Z,NE,EQ,D)                                     FUNC0010
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                               *
C     *                                                               *
C     *                                                               *
C     *    PURPOSE                                                    *
C     *         COMPUTES THE RESIDUALS OF THE DESIGN EQUATIONS        *
C     *                                                               *
C     *    DESCRIPTION OF PARAMETERS                                  *
C     *         Z - UNKNOWN DESIGN PARAMETERS                         *
C     *         EQ - ARRAY OF RESIDUALS                               *
C     *         KZ,NE - NO. OF VARIABLES/EQUATIONS                    *
C     *                                                               *
C     *    USAGE                                                      *
C     *         THIS SUBROUTINE IS CALLED BY THE SUBROUTINE SOLVE(MRQT) *
C     *                                                               *
C     *    SUBROUTINES REQUIRED                                       *
C     *         SUBROUTINE DESIGN                                     *
C     *                                                               *
C     *                                                               *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DOUBLE PRECISION Z                                                FUNC0020
      DIMENSION Z(KZ),EQ(NE),D(NE),P(1,1)                               FUNC0030
      NAZ=1                                                             FUNC0040
      CALL DESIGN(KZ,Z,NE,EQ,P,NAZ)                                     FUNC0050
      RETURN                                                            FUNC0060
      END                                                               FUNC0070
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      SUBROUTINE DERIV(KZ,Z,NE,EQ,P,L)                                  DRIV0010
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *     PURPOSE                                                     *
C     *     COMPUTES THE MATRIX OF FIRST PARTIALS ( OF THE EQUATIONS    *
C     *     WITH RESPECT TO THE VARIABLES )                             *
C     *                                                                 *
C     *     DESCRIPTION OF PARAMETERS                                   *
C     *         Z - UNKNOWN DESIGN PARAMETERS                           *
C     *         EQ - ARRAY OF RESIDUALS                                 *
C     *         KZ,NE - NO. OF VARIABLES/EQUATIONS                      *
C     *         P - MATRIX OF PARTIAL DERIVATIVES                       *
C     *                                                                 *
C     *     USAGE                                                       *
C     *         THIS SUBROUTINE IS CALLED BY THE SUBROUTINE SOLVE(MRQT) *
C     *                                                                 *
C     *     SUBROUTINES REQUIRED                                        *
C     *         SUBROUTINE DESIGN                                       *
C     *                                                                 *
C     *                                                                 *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DOUBLE PRECISION Z                                                DRIV0020
      DIMENSION Z(KZ),EQ(NE),P(NE,KZ)                                   DRIV0030
      NAZ=-1                                                            DRIV0040
      CALL DESIGN(KZ,Z,NE,EQ,P,NAZ)                                     DRIV0050
      RETURN                                                            DRIV0060
      END                                                               DRIV0070
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C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *         TO SYNTHESIZE THE EIGHT LINK MECHANISM FOR TWO RIGID    *
C     *         BODIES GUIDANCE                                         *
C     *                                                                 *
C     *     DESCRIPTION OF PARAMETERS                                   *
C     *         PX - VECTOR OF X-COORDINATES OF THE POINT 'P'           *
C     *         PY - VECTOR OF Y-COORDINATES OF THE POINT 'P'           *
C     *         TH6 - VECTOR OF ANGULAR DISPLACEMENTS OF THE RIGID      *
C     *               BODY CARRYING 'P'                                 *
C     *         SX - VECTOR OF X-COORDINATES OF THE POINT 'S'           *
C     *         SY - VECTOR OF Y-COORDINATES OF THE POINT 'S'           *
C     *         TH3 - VECTOR OF ANGULAR DISPLACEMENTS OF THE RIGID      *
C     *               BODY CARRYING 'S'                                 *
C     *                                                                 *
C     *     USAGE                                                       *
C     *         REQUIRES THE FOLLOWING DATA CARDS                       *
C     *            CARD 1. THE FIVE X-COORDINATES OF P WITH FORMAT      *
C     *                 5F10.0                                          *
C     *            CARD 2. THE FIVE Y-COORDINATES OF P WITH FORMAT      *
C     *                 5F10.0                                          *
C     *            CARD 3. THE FIVE ANGULAR DISPLACEMENTS TH6(NOTE      *
C     *                 TH6(1)=0.0) WITH FORMAT 5F10.0                  *
C     *            CARD 4. THE FIVE X-COORDINATES OF S WITH FORMAT      *
C     *                 5F10.0                                          *
C     *            CARD 5. THE FIVE Y-COORDINATES OF S WITH FORMAT      *
C     *                 5F10.0                                          *
C     *            CARD 6. THE FIVE ANGULAR DISPLACEMENTS TH3(NOTE      *
C     *                 TH3(1)=0.0) WITH FORMAT 5F10.0                  *
C     *                                                                 *
C     *     SUBROUTINES REQUIRED                                        *
C     *         SUBROUTINE  SOLVE                                       *
C     *         SUBROUTINE  COEFF                                       *
C     *         SUBROUTINE  ROTATE                                      *
C     *                                                                 *
C     *                                                                 *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION SA(5),SB(5)                                             TRBG0010
      DIMENSION R(5),THE(5),GAM(5)                                      TRBG0020
      DIMENSION DX(5),DY(5),HX(5),HY(5)                                 TRBG0030
     COMMON NOM                                                         TRBG0040
      DIMENSION GX(5),GY(5)                                             TRBG0050
      DIMENSION FY(5),FX(5),AX(5),AY(5)                                 TRBG0060
      DIMENSION TH3(5),TH2(5),TH1(5),TH5(5),TH6(5),TH8(5),TH11(5),TH10(TRBG0070
     15)                                                                TRBG0080
      REAL MX(5),MY(5),K(5),KL1(4,5),KL2(4,5),KL3(4,5),KL4(4,5),KL5(4,5TRBG0090
     1)                                                                 TRBG0100
      DIMENSION PX(5),PY(5),SX(5),SY(5),X(5),Y(5),QX(5),QY(5),EX(5),EY(TRBG0110
     15),A(5,5),D(5,3)                                                  TRBG0120
      PI=4.*ATAN(1.)                                                    TRBG0130
      ICOUNT=0                                                          TRBG0140
    1 FORMAT(10X,/,I3X,6(F10.5,5X))                                     TRBG0150
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  2   FORMAT(8F10.5)                                                    TRBG0160
  3   FORMAT(5F10.0,I2)                                                 TRBG0170
  4  FORMAT(10X,'DESIGN SPECIFICATIONS',/,17X,'PX',13X,'PY',13X,'ALFA',TRBG0180
    *11X,'SX',13X,'SY',13X,'ETA',/)                                     TRBG0190
  5  FORMAT(5X,'NO SOLUTION FOR THE FIRST LOOP')                        TRBG0200
  6  FORMAT(10X,////)                                                   TRBG0210
  7  FORMAT(10X,'THE FOLLOWING ARE THE SOLUTIONS CORRESPCNDING TC',/,   TRBG0220
    *10X,'F(X,Y)=(',F9.4,',',F9.4,')',5X,'G(X,Y)=(',F9.4,',',F9.4,')',/TRBG0230
    *10X,    'H(X,Y)=(',F9.4,',',F9.4,')',5X,'Q(X,Y)=(',F9.4,',',F9.4,')TRBG0240
    *',/)                                                               TRBG0250
  8  FORMAT( 7X,'M(X,Y)',16X,'A(X,Y)',16X,'B(X,Y)'           ,16X,     TRBG0260
    *'C(X,Y)',16X,'D(X,Y)',16X,'E(X,Y)',/)                              TRBG0270
  9  FORMAT(1X,'(',F8.3,',',F8.3,')',2X,'(',F8.3,',',F8.3,')',2X,'(',   TRBG0280
    * F8.3,',',F8.3,')',2X,'(',F8.3,',',F8.3,')',3X,'(',F8.3,',',F8.3,  TRBG0290
    *')',3X,'(',F8.3,',',F8.3,')')                                      TRBG0300
 10  FORMAT(/,10X,'TOTAL NUMBER OF MECHANISMS=',I4)                     TRBG0310
 11   READ(5,3)  (PX(I),I=1,5),NAZU                                     TRBG0320
      IF(NAZU.GT.0) GO TO 30                                            TRBG0330
      READ(5,2)  (PY(I),I=1,5)                                          TRBG0340
      READ(5,2)  (TH6(I),I=1,5)                                         TRBG0350
      READ(5,2)  (SX(I),I=1,5)                                          TRBG0360
      READ(5,2)  (SY(I),I=1,5)                                          TRBG0370
      READ(5,2)  (TH3(I),I=1,5)                                         TRBG0380
      WRITE(6,4)                                                        TRBG0390
     DO 12 I=1,5                                                        TRBG0400
 12  WRITE(6,20)  PX(I),PY(I),TH6(I),SX(I),SY(I),TH3(I)                 TRBG0410
      DO 13 I=1,5                                                       TRBG0420
      TH3(I)=TH3(I)*PI/180.                                             TRBG0430
      TH6(I)=TH6(I)*PI/180.                                             TRBG0440
      X(I)=0.                                                           TRBG0450
      Y(I)=0.                                                           TRBG0460
      TH1(I)=0.                                                         TRBG0470
      TH10(I)=0.                                                        TRBG0480
 13   CONTINUE                                                          TRBG0490
      CALL COEFF(TH3,TH6,SX,PX,SY,PY,A,D)                               TRBG0500
      CALL SOLVE(A,D,KL3)                                               TRBG0510
      NLOOP3=NOM                                                        TRBG0520
     IF(NOM.EQ.0) WRITE(6,5)                                            TRBG0530
      DO 29  NL3=1,NLOOP3                                               TRBG0540
      DO 4   N=1,5                                                      TRBG0550
      K(N)=KL3(NL3,N)                                                   TRBG0560
 14   CONTINUE                                                          TRBG0570
      HY(1)=K(1)+SY(1)                                                  TRBG0580
      HX(1)=K(2)+SX(1)                                                  TRBG0590
      FX(1)=PX(1)-K(3)                                                  TRBG0600
      FY(1)=PY(1)-K(4)                                                  TRBG0610
      CALL COEFF(TH1,TH3,X,SX,Y,SY,A,D)                                 TRBG0620
      CALL SOLVE(A,D,KL1)                                               TRBG0630
     IF(NOM.EQ.0) GO TO 29                                              TRBG0640
      NLOOP1=NOM                                                        TRBG0650
      DO 28   NL1=1,NLOOP1                                              TRBG0660
      DO 15  N=1,5                                                      TRBG0670
      K(N)=KL1(NL1,N)                                                   TRBG0680
 15   CONTINUE                                                          TRBG0690
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      QY(1)=K(1)                                                        TRBG0700
      QX(1)=K(2)                                                        TRBG0710
      GX(1)=SX(1)-K(3)                                                  TRBG0720
      GY(1)=SY(1)-K(4)                                                  TRBG0730
      WRITE(6,6)                                                        TRBG0740
     WRITE(6,7)   FX(1),FY(1),GX(1),GY(1),HX(1),HY(1),QX(1),QY(1)       TRBG0750
     WRITE(6,8)                                                         TRBG0760
      DO 16  I=1,5                                                      TRBG0770
      QX(I)=QX(1)                                                       TRBG0780
      QY(I)=QY(1)                                                       TRBG0790
 16   CONTINUE                                                          TRBG0800
      XO=GX(1)-QX(1)                                                    TRBG0910
      YO=GY(1)-QY(1)                                                    TRBG0920
      THO=ATAN2(YO,XO)                                                  TRBG0830
      TH2(1)=0.                                                         TRBG0840
      DO 17  I=2,5                                                      TRBG0850
      CALL ROTATE(GX(I),GY(I),GX(1),GY(1),TH3(I),SX(I),SY(I),SX(1),SY(1TRBG0860
     1))                                                                TRBG0870
      XA=GX(I)-QX(1)                                                    TRBG0880
      YA=GY(I)-QY(1)                                                    TRBG0890
      TH2(I)=ATAN2(YA,XA)-THO                                           TRBG0900
 17   CONTINUE                                                          TRBG0910
      CALL COEFF(TH6,TH2,PX,QX,PY,QY,A,D)                               TRBG0920
      CALL SOLVE(A,D,KL2)                                               TRBG0930
      IF(NOM.EQ.0) GO TO 28                                             TRBG0940
      NLOOP2=NOM                                                        TRBG0950
      DO 27  NL2=1,NLOOP2                                               TRBG0960
      DO 18  N=1,5                                                      TRBG0970
     K(N)=KL2(NL2,N)                                                    TRBG0980
 18  CONTINUE                                                           TRBG0990
      EY(1)=K(1)+PY(1)                                                  TRBG1000
      EX(1)=K(2)+PX(1)                                                  TRBG1010
      DX(1)=QX(1)-K(3)                                                  TRBG1020
      DY(1)=QY(1)-K(4)                                                  TRBG1030
      XO=DX(1)-EX(1)                                                    TRBG1040
      YO=DY(1)-EY(1)                                                    TRBG1050
      THO=ATAN2(YO,XO)                                                  TRBG1060
      TH5(1)=0.                                                         TRBG1070
      DO 19  I=2,5                                                      TRBG1080
      CALL ROTATE(DX(I),DY(I),DX(1),DY(1),TH2(I),QX(1),QY(1),QX(1),QY(1TRBG1090
     1))                                                                TRBG1100
      CALL  ROTATE(EX(I),EY(I),EX(1),EY(1),TH6(I),PX(I),PY(I),PX(1),PY(TRBG1110
     11))                                                               TRBG1120
      XA=DX(I)-EX(1)                                                    TRBG1130
      YA=DY(I)-EY(I)                                                    TRBG1140
      TH5(I)=ATAN2(YA,XA)-THO                                           TRBG1150
 19   CONTINUE                                                          TRBG1160
      XO=FX(1)-HX(1)                                                    TRBG1170
      YO=FY(1)-HY(1)                                                    TRBG1180
      THO=ATAN2(YO,XO)                                                  TRBG1190
      TH8(1)=0.                                                         TRBG1200
      DO 20  I=2,5                                                      TRBG1210
      CALL ROTATE(HX(I),HY(I),HX(1),HY(1),TH3(I),SX(I),SY(I),SX(1),SY(1TRBG1220
     1))                                                                TRBG1230
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      CALL ROTATE(FX(I),FY(I),FX(1),FY(1),TH6(I),PX(I),PY(I),PX(1),PY(1TRBG1240
     1))                                                                TRBG1250
      XA=FX(I)-HX(I)                                                    TRBG1260
      YA=FY(I)-HY(I)                                                    TRBG1270
      TH8(I)=ATAN2(YA,XA)-THO                                           TRBG1280
   20 CONTINUE                                                          TRBG1290
      CALL COEFF(TH10,TH8,X,FX,Y,FY,A,D)                                TRBG1300
      CALL SOLVE(A,D,KL4)                                               TRBG1310
     IF(NOM.EQ.0) GO TO 27                                              TRBG1320
      NLOOP4=NOM                                                        TRBG1330
      DO 26  NL4=1,NLOOP4                                               TRBG1340
      DO 21  N=1,5                                                      TRBG1350
      K(N)=KL4(NL4,N)                                                   TRBG1360
   21 CONTINUE                                                          TRBG1370
      MY(1)=K(1)                                                        TRBG1380
      MX(1)=K(2)                                                        TRBG1390
      AX(1)=FX(1)-K(3)                                                  TRBG1400
      AY(1)=FY(1)-K(4)                                                  TRBG1410
      XO=AX(1)-MX(1)                                                    TRBG1420
      YO=AY(1)-MY(1)                                                    TRBG1430
      THO=ATAN2(YO,XO)                                                  TRBG1440
      TH11(1)=0.                                                        TRBG1450
      DO 22  I=2,5                                                      TRBG1460
      CALL ROTATE(AX(I),AY(I),AX(1),AY(1),TH8(I),HX(I),HY(1),HX(1),HY(1TRBG1470
     1))                                                                TRBG1480
      XA=AX(I)-MX(1)                                                    TRBG1490
      YA=AY(I)-MY(1)                                                    TRBG1500
      TH11(I)=ATAN2(YA,XA)-THO                                          TRBG1510
   22 CONTINUE                                                          TRBG1520
      DO 23    I=2,5                                                    TRBG1530
      MX(I)=MX(1)                                                       TRBG1540
      MY(I)=MY(1)                                                       TRBG1550
   23 CONTINUE                                                          TRBG1560
      CALL COEFF(TH11,TH5,MX,EX,MY,EY,A,D)                              TRBG1570
      CALL SOLVE(A,D,KL5)                                               TRBG1580
      IF(NOM.EQ.0) GO TO 26                                             TRBG1590
      NLOOP5=NOM                                                        TRBG1600
      DO 25  NL5=1,NLOOP5                                               TRBG1610
      DO 24  N=1,5                                                      TRBG1620
      K(N)=KL5(NL5,N)                                                   TRBG1630
   24 CONTINUE                                                          TRBG1640
      BY=K(1)+MY(1)                                                     TRBG1650
      BX=K(2)+MX(1)                                                     TRBG1660
      CX=EX(1)-K(3)                                                     TRBG1670
      CY=EY(1)-K(4)                                                     TRBG1680
      WRITE(6,9)  MX(1),MY(1),AX(1),AY(1),BX,BY,CX,CY,DX(1),DY(1),EX(1)TRBG1690
     *,EY(1)                                                            TRBG1700
     ICOUNT=ICOUNT+1                                                    TRBG1710
   25 CONTINUE                                                          TRBG1720
   26 CONTINUE                                                          TRBG1730
   27 CONTINUE                                                          TRBG1740
   28 CONTINUE                                                          TRBG1750
   29 CONTINUE                                                          TRBG1760
      WRITE(6,10) ICOUNT                                                TRBG1770
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      GO TO 11                                                          TRBG1780
   30 STOP                                                              TRBG1790
      END                                                               TRBG1800
```

```
      SUBROUTINE SOLVE(A,D,DK)                                          LNSP0010
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *    PURPOSE                                                      *
C     *         SOLUTION OF LINEAR EQUATIONS AND TO DETERMINE THE       *
C     *         K'S INVOLVED IN LINEAR SUPERPOSITION                    *
C     *                                                                 *
C     *    DESCRIPTION OF PARAMETERS                                    *
C     *         A - THE MATRIX OF COEFFICIENTS OF THE LINEAR EQUATIONS  *
C     *         D - THE MATRIX WHOSE COLUMNS CONTAIN THE RIGHT HAND     *
C     *                SIDES OF THE EQUATIONS                           *
C     *         DK - THE ARRAY OF THE K'S                               *
C     *         NOM- COUNT OF REAL SOLUTIONS                            *
C     *                                                                 *
C     *    USAGE                                                        *
C     *         THIS SUBROUTINE IS CALLED BY THE MAIN PROGRAM FOR       *
C     *         THE DESIGN PROBLEMS INVOLVING LINEAR SUPERPOSITION      *
C     *                                                                 *
C     *    SUBROUTINES REQUIRED                                         *
C     *         SUBROUTINE  MATINV                                      *
C     *         SUBROUTINE  POLRT                                       *
C     *                                                                 *
C     *                                                                 *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION D(5,3)                                                  LNSP0020
C                                                                       LNSPCCCC
      REAL K(5),L(5),M(5),N(5),LEM1,LEM2                                LNSP0030
      DIMENSION DK(4,5)                                                 LNSP0040
      COMMON NOM                                                        LNSP0050
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      DIMENSION R(5),T(5),G(5),A(5,5),B(5,1),XCOF(5),ROOTR(5),ROOTI(5) LNSP0060
      DIMENSION COF(5)                                                  LNSP0070
      DIMENSION C(5,5)                                                  LNSP0080
      DO  1 IK=1,5                                                      LNSP0090
      DO  1 IJ=1,5                                                      LNSP0100
      C(IK,IJ)=A(IK,IJ)                                                 LNSP0110
      B(IJ,1)=D(IJ,1)                                                   LNSP0120
    1 CONTINUE                                                          LNSP0130
      CALL MATINV(C,5,5,B,1,1)                                          LNSP0140
      DO  2 J=1,5                                                       LNSP0150
      N(J)=B(J,1)                                                       LNSP0160
      B(J,1)=D(J,2)                                                     LNSP0170
    2 CONTINUE                                                          LNSP0180
      DO  3 IJ=1,5                                                      LNSP0190
      DO  3 IK=1,5                                                      LNSP0200
      C(IK,IJ)=A(IK,IJ)                                                 LNSP0210
    3 CONTINUE                                                          LNSP0220
      CALL MATINV(C,5,5,B,1,1)                                          LNSP0230
      DO   4 J=1,5                                                      LNSP0240
      L(J)=B(J,1)                                                       LNSP0250
      B(J,1)=D(J,3)                                                     LNSP0260
    4 CONTINUE                                                          LNSP0270
      DO  5 IK=1,5                                                      LNSP0280
      DO  5 IJ=1,5                                                      LNSP0290
      C(IK,IJ)=A(IK,IJ)                                                 LNSP0300
    5 CONTINUE                                                          LNSP0310
      CALL MATINV(C,5,5,B,1,1)                                          LNSP0320
      DO   6 J=1,5                                                      LNSP0330
      M(J)=B(J,1)                                                       LNSP0340
    6 CONTINUE                                                          LNSP0350
      A1=L(2)*L(3)+L(1)*L(4)                                            LNSP0360
      A2=L(2)*M(3)+M(2)*L(3)+L(1)*M(4)+M(1)*L(4)                        LNSP0370
      A3=N(3)*L(2)+N(2)*L(3)+L(1)*N(4)+N(1)*L(4)-1.                     LNSP0380
      A4=M(1)*M(4)+M(2)*M(3)                                            LNSP0390
      A5=N(2)*M(3)+M(2)*N(3)+N(1)*M(4)+M(1)*N(4)                        LNSP0400
      A6=N(2)*N(3)+N(1)*N(4)                                            LNSP0410
      B1=L(1)*L(3)-L(2)*L(4)                                            LNSP0420
      B2=L(1)*M(3)+M(1)*L(3)-L(2)*M(4)-M(2)*L(4)                        LNSP0430
      B3=L(1)*N(3)+N(1)*L(3)-L(2)*N(4)-N(2)*L(4)                        LNSP0440
      B4=M(1)*M(3)-M(2)*M(4)                                            LNSP0450
      B5=N(1)*M(3)+M(1)*N(3)-N(2)*M(4)-M(2)*N(4)-1.                     LNSP0460
      B6=N(1)*N(3)-N(2)*N(4)                                            LNSP0470
      C1=A2*B2                                                          LNSP0480
      C2=A2*B3+B2*A3                                                    LNSP0490
      C3=A3*B3                                                          LNSP0500
      D1=B2**2-B1*B4                                                    LNSP0510
      D2=2.*B2*B3-B1*B5                                                 LNSP0520
      D3=B3**2-B1*B6                                                    LNSP0530
      E1=C1*B4                                                          LNSP0540
      E2=C2*B4+C1*B5                                                    LNSP0550
      E3=C3*B4+C1*B6+C2*B5                                              LNSP0560
      E4=C2*B6+C3*B5                                                    LNSP0570
      E5=C3*B6                                                          LNSP0580
      F1=D1*A4                                                          LNSP0590
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      F2=D2*A4+D1*A5                                                    LNSP0600
      F3=D3*A4+D1*A6+D2*A5                                              LNSP0610
      F4=D2*A6+D3*A5                                                    LNSP0620
      F5=D3*A6                                                          LNSP0630
      G1=A2*B2                                                          LNSP0640
      G2=B2*A3+A2*B3                                                    LNSP0650
      G3=A3*B3                                                          LNSP0660
      H1=A2**2-A1*A4                                                    LNSP0670
      H2=2.*A2*A3-A1*A5                                                 LNSP0680
      H3=A3**2-A1*A6                                                    LNSP0690
      P1=G1*A4                                                          LNSP0700
      P2=G2*A4+G1*A5                                                    LNSP0710
      P3=G3*A4+G1*A6+G2*A5                                              LNSP0720
      P4=G2*A6+G3*A5                                                    LNSP0730
      P5=G3*A6                                                          LNSP0740
      Q1=H1*B4                                                          LNSP0750
      Q2=H2*B4+H1*B5                                                    LNSP0760
      Q3=H3*B4+H1*B6+H2*B5                                              LNSP0770
      Q4=H2*B6+H3*B5                                                    LNSP0780
      Q5=H3*B6                                                          LNSP0790
      P1=P1*B1                                                          LNSP0800
      P2=P2*B1                                                          LNSP0810
      P3=P3*B1                                                          LNSP0820
      P4=P4*B1                                                          LNSP0830
      P5=P5*B1                                                          LNSP0840
      Q1=Q1*B1                                                          LNSP0850
      Q2=Q2*B1                                                          LNSP0860
      Q3=Q3*B1                                                          LNSP0870
      Q4=Q4*B1                                                          LNSP0880
      Q5=Q5*B1                                                          LNSP0890
      F1=F1*A1                                                          LNSP0900
      F2=F2*A1                                                          LNSP0910
      F3=F3*A1                                                          LNSP0920
      F4=F4*A1                                                          LNSP0930
      F5=F5*A1                                                          LNSP0940
      E1=E1*A1                                                          LNSP0950
      E2=E2*A1                                                          LNSP0960
      E3=E3*A1                                                          LNSP0970
      E4=E4*A1                                                          LNSP0980
      E5=E5*A1                                                          LNSP0990
      XCOF(5)=(A1**2)*(B4**2)+(B1**2)*(A4**2)+F1+Q1-E1-P1               LNSP1000
      XCOF(4)=2.*(A1**2)*B4*B5+2.*(B1**2)*A5*A4+F2+Q2-E2-P2             LNSP1010
      XCOF(3)=(A1**2)*(B5**2)+2.*(A1**2)*(B4*B6)+(B1**2)*(A5**2)+2.*(B1LNSP1020
     1**2)*A4*A6+F3+Q3-E3-P3                                            LNSP1030
      XCOF(2)=2.*(A1**2)*B5*B6+2.*(B1**2)*A5*A6+F4+Q4-E4-P4             LNSP1040
      XCOF(1)=(A1**2)*(B6**2)+(B1**2)*(A6**2)+F5+Q5-E5-P5               LNSP1050
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)                            LNSP1060
      NOM=0                                                             LNSP1070
      DO  12  IM=1,4                                                    LNSP1080
      IF(ROOTI(IM).NE.0.)   GO TO 12                                    LNSP1090
      LEM2=ROOTR(IM)                                                    LNSP1100
      Y=LEM2*A2+A3                                                      LNSP1110
      Z=((LEM2)**2)*A4+LEM2*A5+A6                                       LNSP1120
      X=A1                                                              LNSP1130
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      S=Y**2-4.*A1*Z                                                    LNSP1140
      IF (S) 12,7,7                                                     LNSP1150
    7 S=SQRT(S)                                                         LNSP1160
      XL1=(-Y+S)/(2.*A1)                                                LNSP1170
      XL2=(-Y-S)/(2.*A1)                                                LNSP1180
      YB=LEM2*B2+B3                                                     LNSP1190
      ZB=((LEM2**2)*B4+LEM2*B5+B6)                                      LNSP1200
      XB=B1                                                             LNSP1210
      SB=YB**2-4.*B1*ZB                                                 LNSP1220
      IF (SB) 12,8,8                                                    LNSP1230
    8 SB=SQRT(SB)                                                       LNSP1240
      XLB1=(-YB+SB)/(2.*B1)                                             LNSP1250
      XLB2=(-YB-SB)/(2.*B1)                                             LNSP1260
      AXL1=ABS(XL1-XLB1)                                                LNSP1270
      AXL2=ABS(XL1-XLB2)                                                LNSP1280
      IF(AXL1.LE.AXL2)  AXL=XLB1                                        LNSP1290
      IF(AXL1.LE.AXL2)  AB=AXL1                                         LNSP1300
      IF(AXL2.LE.AXL1) AXL=XLB2                                         LNSP1310
      IF(AXL2.LE.AXL1) AB=AXL2                                          LNSP1320
      AAXL1=ABS(XL2-XLB1)                                               LNSP1330
      AAXL2=ABS(XL2-XLB2)                                               LNSP1340
      IF(AAXL1.LE.AAXL2)  BXL=XLB1                                      LNSP1350
      IF(AAXL1.LE.AAXL2)          ABB=AAXL1                             LNSP1360
      IF(AAXL2.LE.AAXL1) BXL=XLB2                                       LNSP1370
      IF(AAXL2.LE.AAXL1)  ABB=AAXL2                                     LNSP1380
      CONTINUE                                                          LNSP1390
      IF(AXL1.GT.0.1000.AND.AXL2.GT.0.1000.AND.AAXL1.GT.0.1000.AND.AAXLLNSP1400
     12.GT.0.1000) GO TO 12                                             LNSP1410
      IF(AB.GE.ABB) GO TO 9                                             LNSP1420
      LEM1= AXL                                                         LNSP1430
      GO TO 10                                                          LNSP1440
    9 CONTINUE                                                          LNSP1450
      LEM1=BXL                                                          LNSP1460
   10 CONTINUE                                                          LNSP1470
      NOM=NOM+1                                                         LNSP1480
      DO 11 I=1,5                                                       LNSP1490
      K(I)= N(I)+LEM1*L(I)+LEM2*M(I)                                    LNSP1500
      DK(NOM,I)=K(I)                                                    LNSP1510
   11 CONTINUE                                                          LNSP1520
      AX1=SQRT(K(3)**2+K(4)**2)                                         LNSP1530
      AX3=SQRT(K(1)**2+K(2)**2)                                         LNSP1540
      AX2=SQRT(K(5)+AX1**2+AX3**2)                                      LNSP1550
   12 CONTINUE                                                          LNSP1560
      RETURN                                                            LNSP1570
      END                                                               LNSP1580
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      SUBROUTINE COEFF(G,TH,P1X,P2X,P1Y,P2Y,A,D)                        COEFF001
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *    PURPOSE                                                      *
C     *        TO COMPUTE THE COEFFICIENTS OF THE LINEARIZED EQUAT-     *
C     *        IONS BY KNOWING THE MOTIONS OF TWO RIGID BODIES          *
C     *                                                                 *
C     *    DESCRIPTION OF PARAMETERS                                    *
C     *        G - ARRAY OF THE ANGULAR  DISPLACEMENTS OF THE RIGID     *
C     *        BODY CARRYING THE POINT P1(P1X,P1Y)                      *
C     *        TH- ARRAY OF THE ANGULAR  DISPLACEMENTS OF THE RIGID     *
C     *        BODY CARRYING THE POINT P2(P2X,P2Y)                      *
C     *        A - THE TWO DIMENSIONAL ARRAY OF COEFFICIENTS RETURNED   *
C     *            BY THE PROGRAM                                       *
C     *        D - THE TWO DIMENSIONAL ARRAY OF THE RIGHT HAND SIDE     *
C     *            OF THE EQUATIONS                                     *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     *                                                                 *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION G(5),TH(5),P1X(5),P2X(5),P1Y(5),P2Y(5),A(5,5),D(5,3)    COEFF002
      DO 10 I=1,5                                                       COEFF003
      CG=COS(G(I))                                                      COEFF004
      SG=SIN(G(I))                                                      COEFF005
      ST=SIN(TH(I))                                                     COEFF006
      CT=COS(TH(I))                                                     COEFF007
      PX=P2X(I)-P1X(I)                                                  COEFF008
      PY=P2Y(I)-P1Y(I)                                                  COEFF009
      A(I,2)=2.*(PX*CG+PY*SG)                                           COEFF010
      A(I,1)=-2.*(PX*SG-PY*CG)                                          COEFF011
      A(I,4)=-2.*(PX*ST-PY*CT)                                          COEFF012
      A(I,3)=2.*(PX*CT+PY*ST)                                           COEFF013
      A(I,5)=1.0                                                        COEFF014
      D(I,1)=PX*PX+PY*PY                                                COEFF015
      D(I,2)=2.*(CT*CG+ST*SG)                                           COEFF016
      D(I,3)=2.*(CG*ST-SG*CT)                                           COEFF017
   10 CONTINUE                                                          COEFF018
      RETURN                                                            COEFF019
      END                                                               COEFF020
      SUBROUTINE ROTATE(DXN,DYN,DX,DY,G,BXN,BYN,BX,BY)                  ROTATE01
      DXN=DX*COS(G)-DY*SIN(G)+BXN-BX*COS(G)+BY*SIN(G)                   ROTATE02
      DYN=DX*SIN(G)+DY*COS(G)+BYN-BX*SIN(G)-BY*COS(G)                   ROTATE03
      RETURN                                                            ROTATE04
      END                                                               ROTATE05
      SUBROUTINE MATINV (A,NN,N,B,MM,M)                                 MATIN001
     DIMENSION A(NN,NN),B(NN,MM),IPIVOT(50),PIVOT(50),INDEX(50,2)       MATIN002
C    INITIALIZATION                                                     MATIN003
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)             MATIN004
   15 DO 20 J=1,N                                                       MATIN005
   20 IPIVOT(J)=0                                                       MATIN006
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   30 DO 550 I=1,N                                                      MATIN007
   40 AMAX=0.0                                                          MATIN008
   45 DO 105 J=1,N                                                      MATIN009
   50 IF (IPIVOT(J)-1)60,105,60                                         MATIN010
   60 DO 100 K=1,N                                                      MATIN011
   70 IF(IPIVOT(K)-1)80,100,740                                         MATIN012
   80 IF (ABS(AMAX)-ABS(A(J,K)))85,100,100                              MATIN013
   85 IROW=J                                                            MATIN014
   90 ICOLUM=K                                                          MATIN015
   95 AMAX=A(J,K)                                                       MATIN016
  100 CONTINUE                                                          MATIN017
  105 CONTINUE                                                          MATIN018
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1                                   MATIN019
  130 IF(IROW-ICOLUM)140,260,140                                        MATIN020
  140 CONTINUE                                                          MATIN021
  150 DO 200 L=1,N                                                      MATIN022
  160 SWAP=A(IROW,L)                                                    MATIN023
  170 A(IROW,L)=A(ICOLUM,L)                                             MATIN024
  200 A(ICOLUM,L)=SWAP                                                  MATIN025
  205 IF(M)260,260,210                                                  MATIN026
  210 DO 250 L=1,M                                                      MATIN027
  220 SWAP=B(IROW,L)                                                    MATIN028
  230 B(IROW,L)=B(ICOLUM,L)                                             MATIN029
  250 B(ICOLUM,L)=SWAP                                                  MATIN030
  260 INDEX(I,1)=IROW                                                   MATIN031
  270 INDEX(I,2)=ICOLUM                                                 MATIN032
  310 PIVOT(I)=A(ICOLUM,ICOLUM)                                         MATIN033
  330 A(ICOLUM,ICOLUM)=1.0                                              MATIN034
  340 DO 350 L=1,N                                                      MATIN035
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)                                  MATIN036
  355 IF(M)380,380,360                                                  MATIN037
  360 DO 370 L=1,M                                                      MATIN038
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)                                  MATIN039
  380 DO 550 L1=1,N                                                     MATIN040
  390 IF(L1-ICOLUM)400,550,400                                          MATIN041
  400 T=A(L1,ICOLUM)                                                    MATIN042
  420 A(L1,ICOLUM)=0.0                                                  MATIN043
  430 DO 450 L=1,N                                                      MATIN044
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T                                     MATIN045
  455 IF(M)550,550,460                                                  MATIN046
  460 DO 500L=1,M                                                       MATIN047
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T                                     MATIN048
  550 CONTINUE                                                          MATIN049
  600 DO 710 I=1,N                                                      MATIN050
  610 L=N+1-I                                                           MATIN051
  620 IF (INDEX(L,1)-INDEX(L,2))630,710,630                             MATIN052
  630 JROW=INDEX(L,1)                                                   MATIN053
  640 JCOLUM=INDEX(L,2)                                                 MATIN054
  650 DO 705 K=1,N                                                      MATIN055
  660 SWAP=A(K,JROW)                                                    MATIN056
  670 A(K,JROW)=A(K,JCOLUM)                                             MATIN057
  700 A(K,JCOLUM)=SWAP                                                  MATIN058
  705 CONTINUE                                                          MATIN059
  710 CONTINUE                                                          MATIN060
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  740 RETURN                                                            MATIN061
      END                                                               MATIN062
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      SUBROUTINE SOLVE(K,N,I,ICON,KERR,B,BMIN,BMAX,Y,Z,PH,SPAR,QI,DE,NO,MRQT0010
     *NITER,DSIG)                                                        MRQT0020
C     SUBROUTINE SOLVE - 2-6-69 - P W TIDWELL
C
C     K = THE NUMBER OF UNKNOWNS TO BE DETERMINED
C     N = THE NUMBER OF OBSERVATIONS OF Y AVAILABLE
C     I = ITERATION COUNT - INITIALLY SHOULD BE ZERO
C     ICON = THE NUMBER OF B VALUES NOT SATISFYING CONVERGENCE REQUIRMT
C     ICON=-4 SINGULAR MATRIX
C     *                                                                 *
      DOUBLE PRECISION PH,BMIN, BI, SSQ,QI,Q,FK, GR, PHL, B,DE,BMAX     MRQT0030
C
C     ON RETURN TO THE MAIN PROGRAM
C     ICON = 0 INDICATES FINAL SOLUTION OF PROBLEM
C     ICON = -1 INDICATES MORE UNKNOWNS THAN FUNCTIONS
C     ICON = -2 INDICATES TOTAL VARIABLES ARE ZERO
C     ICON = -3 TOO MANY ITERATIONS
C
C     KERR IS AN ERROR FLAG WHICH INDICATES THAT THE MATRIX OF NORMAL
C      EQUATIONS IS SINGULAR
C
C     B = A K DIMENSIONAL VECTOR OF THE UNKNOWNS
C     BMIN(I) = MINIMUM VALUE FOR B(I) TO BE ALLOWED (INPUT)
C     BMAX(I) = MAXIMUM VALUE FOR B(I) TO BE ALLOWED (INPUT)
C     Y = AN N DIMENSIONAL VECTOR OF GIVEN VALUES (OBSERVATIONS)
C     Z = AN N DIMENSIONAL VECTOR OF COMPUTED VALUES
C     PH = FUNCTIONAL VALUE ( SUM OF THE SQUARES OF  (Y - Z) )
C
C     SPAR IS A VECTOR WHICH SUPPLIES THE SUBROUTINE WITH THE
C     FOLLOWING FOUR PARAMETERS
C
      DOUBLE PRECISION DABS,DSQRT,X,DET,SUM,XSAV                        MRQT0040
C     FNU = THE NU FACTOR GIVEN IN THE SOURCE PAPER
C     FLA = THE LAMBDA FACTOR CITED IN SOURCE PAPER
C     EPS1 = CONVERGENCE CRITERIA FOR THE RESIDUAL SUM OF SQUARES
C     EPS2 = CONVERGENCE CRITERIA FOR THE PARAMETERS
C
C
C     THE PROGRAM WILL NOT RUN WITH BOTH EPS1 AND EPS2 SET TO 0.0
C
C     CONVERGENCE CRITERIA OF THE PARAMETERS IS AS FOLLOWS
C     IF( B(J),(I+1) -B(J),(I) /(1.0E-20 + B(J),(I) ) - EPS2= 0.0),
C     FOR ALL OF THE J = 1,2,...,K PARAMETERS OF THE I+1 ST ITERATION,
C       THEN CONVERGENCE HAS BEEN REACHED ON THE PARAMETERS
C
C     CONVERGENCE CRITERIA ON THE RESIDUAL SUM OF SQUARES IS AS FOLLOWS
C     IF((RSS(I) - RSS(I+1))/RSS(I) - EPS1 =  0.0), THEN CONVERGENCE ON
C     THE RESIDUAL SUM OF SQUARES HAS BEEN REACHED ON THE I+1 ST
C     ITERATION
C
C     DIMENSIONS OF MATRICES ARE
C      B(K),Z(N),Y(N),...
C     P(N,K)...,SPAR(4)
C
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C     SOURCE PAPER BY D W MARQUARDT, J.SIAM. JUNE 1963, P. 431
C
C
C     DIMENSION B(1),Z(1),Y(1),BMIN(1),BMAX(1),QI(8,8),SPAR(1),DE(1)
C     DIMENSION P(100,8),BI(8),ZI(100),Q(8,8),X(8),GR(8)
C
C     DATA LDIMQ/8/,LDIMP/100/
      DIMENSION Z(K),B(K),Y(N),BMIN(K),BMAX(K),QI(N,K)                  MRQT0050
      DIMENSION P(42,42),BI(42),ZI(42),Q(42,42),X(42),GR(42)            MRQT0060
     *  ,DE(42),SPAR(4),XSAV(42)                                        MRQT0070
      DATA LDIMQ/42/,LDIMP/42/                                          MRQT0080
C
C     LDIMQ IS THE FIRST DIMENSION OF THE ARRAYS Q AND QI.
C
C
C
C     THE VECTOR QI WILL CONTAIN ON RETURN WITH ICON ZERO OR
C     I = 30 THE INVERSE OF THE MATRIX OF NORMAL EQUATIONS
C
C
      COSGAM=0.9                                                        MRQT0090
      COSGAM=0.98                                                       MRQT0100
      KERR=0                                                            MRQT0110
      IF(K) 140,140,10                                                  MRQT0120
   10 IF(I) 40,40,20                                                    MRQT0130
   20 IF(I - NITER) 160,30,30                                           MRQT0140
   30 WRITE (NO,1) NITER                                                MRQT0150
      ICON=-3                                                           MRQT0160
      RETURN                                                            MRQT0170
    1 FORMAT('1MORE THAN',I3,'ITERATIONS ATTEMPTED')                    MRQT0180
C     NOW INITIALIZE SUBROUTINE THE FIRST TIME CALLED (I = 0)
C
C
   40 FNU=SPAR(1)                                                       MRQT0190
   80 FLA = SPAR(2)                                                     MRQT0200
      IF(ABS(SPAR(3)) + ABS(SPAR(4))) 82,81,82                          MRQT0210
   81 WRITE (NO,2)                                                      MRQT0220
      RETURN                                                            MRQT0230
    2 FORMAT(1H1,'CONVERGENCE CRITERIA OF EPS1 OR EPS2 NOT DEFINED'//)  MRQT0240
   82 WRITE (NO,83) SPAR(3),SPAR(4)                                     MRQT0250
   83 FORMAT(//10H SPAR(3) = E15.7,5X10H SPAR(4) = E15.7//)             MRQT0260
      MAXSUB=10                                                         MRQT0270
      IF(SPAR(3))90,100,100                                             MRQT0280
   90 SPAR(3) = 0.00001                                                 MRQT0290
  100 EPS2 = SPAR(3)                                                    MRQT0300
      IF(SPAR(4)) 110,120,120                                           MRQT0310
  110 SPAR(4) = 0.00001                                                 MRQT0320
  120 EPS1 = SPAR(4)                                                    MRQT0330
      L = 1                                                             MRQT0340
      DO 125 J = 1,N                                                    MRQT0350
      Z(J) = 0.0                                                        MRQT0360
  125 ZI(J) = 0.0                                                       MRQT0370
C
C     INITIALIZATION NOW COMPLETED
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C
      GO TO 170                                                         MRQT0380
  140 ICON = -2                                                         MRQT0390
  150 SPAR(2) = FLA                                                     MRQT0400
      I = I + 1                                                         MRQT0410
      RETURN                                                            MRQT0420
  160 L = 2                                                             MRQT0430
  170 DO 450 I1 = L,2                                                   MRQT0440
      GO TO (180,210), I1                                               MRQT0450
C
C     THE ROUTE THROUGH 180 TO 200 AND THENCE TO 450 IS ONLY TAKEN THE
C     FIRST TIME THAT THE SUBROUTINE IS CALLED
C
  180 DO 190 J = 1,K                                                    MRQT0460
  190 BI(J) = B(J)                                                      MRQT0470
      CALL FUNC(K,B,N,Z,Y)                                              MRQT0480
      PH = 0.0                                                          MRQT0490
C
C     NOW COMPUTE THE NORMAL EQUATIONS
C
      DO 200 J = 1,N                                                    MRQT0500
  200 PH=PH+(Y(J)-Z(J))**2/DSIG**2                                      MRQT0510
      GO TO 450                                                         MRQT0520
  210 CALL DERIV(K,B,N,Z,P,LDIMP)                                       MRQT0530
      DO 250 JA=1,K                                                     MRQT0540
      SUM=0.                                                            MRQT0550
      DO 230 J=1,N                                                      MRQT0560
  230 SUM=SUM+(Y(J)-Z(J))*P(J,JA)                                       MRQT0570
      XSAV(JA)=SUM/DSIG**2                                              MRQT0580
      DO 250 JB=1,JA                                                    MRQT0590
      SUM=0.                                                            MRQT0600
      DO 240 J=1,N                                                      MRQT0610
  240 SUM=SUM+P(J,JA)*P(J,JB)                                           MRQT0620
      QI(JA,JB)=SUM/DSIG**2                                             MRQT0630
  250 QI(JB,JA)=QI(JA,JB)                                               MRQT0640
C     WRITE(6,3)(XSAV(J),J=1,K)                                         MRQTCCCC
C   3 FORMAT(10X'GRADIENT OF SUM OF SQUARES'/(1X5D15.5))                MRQTCCCC
C
C     MATRIX OF NORMAL EQUATIONS NOW COMPUTED AND RESIDING IN THE
C     QI ARRAY
C
C     NOW START THE PROCEDURE TO ESTIMATE THE PARAMETERS
C
      NSUB=0                                                            MRQT0650
      FLA = FLA/FNU                                                     MRQT0660
      WRITE(6,5) I,FLA                                                  MRQT0670
    5 FORMAT(10X,'ITERATION NO.=',I3,5X,'FLA=',E12.5)                   MRQT0680
  300 DO 310 J=1,K                                                      MRQT0690
      DO 310 JJ=1,K                                                     MRQT0700
  310 Q(J,JJ)=QI(J,JJ)                                                  MRQT0710
C
C     MOVE THE MATRIX OF NORMAL EQUATIONS INTO THE Q ARRAY
C     THIS WILL PRESERVE THE ORIGINAL MATRIX FOR USE IN THE EVENT
C     THAT A DIFFERENT VALUE OF FLA NEEDS TO BE USED OR THAT
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C     CONVERGENCE HAS BEEN OBTAINED AND THE INVERSE NEEDS TO BE
C     COMPUTED BEFORE RETURN TO THE CALLING PROGRAM
C
C
C     NOW LOCATE THE DIAGONAL ELEMENTS, TAKE THEIR SQUARE ROOTS
C     AND SAVE THEM TO USE IN SCALING THE MATRIX
C
      DO 320 J = 1,K                                                    MRQT0720
  320 DE(J)=DSQRT(Q(J,J))                                               MRQT0730
C
C     THE DE ARRAY CONTAINS THE MATRIX SCALING FACTORS
C
C     NOW SCALE THE MATRIX AND SAVE THE GRADIENT VECTOR TO USE LATER
C     IN COMPUTING THE ANGLE BETWEEN THE GRADIENT AND THE
C     GAUSS-NEWTON-MARQUARDT VECTOR
C
      DO 340 J=1,K                                                      MRQT0740
      X(J)=XSAV(J)/DE(J)                                                MRQT0750
      GR(J)=X(J)                                                        MRQT0760
      DO 340 JJ=1,K                                                     MRQT0770
  340 Q(J,JJ)=Q(J,JJ)/(DE(J)*DE(JJ))                                    MRQT0780
C
C     NORMAL EQUATIONS NOW SCALED.  THE GR ARRAY CONTAINS THE
C     GRADIENT VECTOR
C
C     NOW ADD FLA TO THE K DIAGONAL ELEMENTS
C
      IF(FLA.EQ.0.000)GO TO 351                                         MRQT0790
      DO 350 J1 = 1,K                                                   MRQT0800
  350 Q(J1,J1)=Q(J1,J1)+FLA                                             MRQT0810
C
C     NOW SOLVE FOR THE CORRECTIONS FOR THE PARAMETERS
C
  351 M=1                                                               MRQT0820
      CALL MATINV(Q,K,X,M,DET,LDIMQ)                                    MRQT0830
      IF(M.GT.0.OR.DET.LE.0.) GO TO 7                                   MRQT0840
      GO TO 360                                                         MRQT0850
    7 WRITE(6,8) DET ,M                                                 MRQT0860
    8 FORMAT(5X,'DET.LT.0  AND =',D15.7 ,' OR M = ',I5)                 MRQT0870
      GO TO 440                                                         MRQT0880
C
C     NORMAL EQUATIONS SOLVED.  CORRECTION VECTOR OCCUPIES THE FIRST
C     K ELEMENTS OF THE Q ARRAY.  IF A SINGULAR MATRIX IS ENCOUNTERED
C     THE RETURN FROM MATINV WILL HAVE KERR NOT ZERO. ABORT THE
C     PROBLEM IN THIS EVENT.
C
C     NEXT THING DONE IS TO COMPUTE THE COSINE OF THE ANGLE BETWEEN
C     THE TWO VECTORS AND SAVE FOR LATER USE
C
  360 SA = 0.0                                                          MRQT0890
      SB = 0.0                                                          MRQT0900
      SC = 0.0                                                          MRQT0910
      DO 370 J = 1,K                                                    MRQT0920
      SA=SA+X(J)*GR(J)                                                  MRQT0930
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      SB=SB+X(J)**2                                                     MRQT0940
  370 SC = SC + GR(J)**2                                                MRQT0950
      ANGLE =      SA/(SQRT(SB*SC))                                     MRQT0960
C
C     NEXT INITIALIZE A SCALE FACTOR TO BE USED IN THE EVENT THE
C     ANGLE BETWEEN THE VECTORS IS GREATER THAN 30 DEGREES
C     (THE COSINE OF THE ANGLE LESS THAN 0.866)
C
      FK = 1.0                                                          MRQT0970
C
C     INCREMENT BI AND CHECK FOR ANY CONSTRAINTS BEING VIOLATED.
C
  380 DO 400 J = 1,K                                                    MRQT0980
      BI(J)=B(J)+FK*X(J)/DE(J)                                          MRQT0990
      IF(BI(J)-BMIN(J))405,390,390                                      MRQT1000
  390 IF(BI(J)-BMAX(J))400,400,405                                      MRQT1010
  400 CONTINUE                                                          MRQT1020
      GO TO 407                                                         MRQT1030
  405 NSUB=NSUB+1                                                       MRQT1040
      IF(NSUB-MAXSUB)430,430,413                                        MRQT1050
C
C     NEXT CHECK FOR A REDUCTION OF THE RESIDUAL SUM OF SQUARES.
C
  407 CALL FUNC(K,BI,N,ZI,Y)                                            MRQT1060
      SSQ = 0.0                                                         MRQT1070
      DO 410 J = 1,N                                                    MRQT1080
  410 SSQ=SSQ+(Y(J)-ZI(J))**2/DSIG**2                                   MRQT1090
      WRITE (NO,411) I,PH,SSQ,ANGLE,FK,FLA,EPSI,EPS2                    MRQT1100
  411 FORMAT(/10H ITERATION I3,5X4HPH =D16.8,5X5HSSQ =D16.8,5X7HANGLE = MRQT1110
     X   E11.3,5X4HFK =D11.3,5X5HFLA =E11.3/40X6HEPS1 =E11.3,5X6HEPS2 = MRQT1120
     X   E11.3/1H )                                                     MRQT1130
      IF(SSQ-PH)450,415,415                                             MRQT1140
  415 NSUB=NSUB+1                                                       MRQT1150
      IF(NSUB-MAXSUB)412,412,413                                        MRQT1160
  413 WRITE (NO,414) MAXSUB                                             MRQT1170
  414 FORMAT(//20H MORE THAN MAXSUB = I3,24H SUBITERATICNS REQUIRED.    MRQT1180
     X   //)                                                            MRQT1190
      ICON=-3                                                           MRQT1200
      GO TO 720                                                         MRQT1210
  412 CONTINUE                                                          MRQT1220
C
C     NOW SEE IF THE NEW ESTIMATES HAVE REDUCED THE RESIDUAL SUM OF
C     SQUARES.  IF NOT, AND THE ANGLE IS LESS THAN 30 DEGREES, ADD ONLY
C     1/2 OF THE CORRECTION PREVIOUSLY USED AND RECOMPUTE THE
C     RESIDUAL SUM OF SQUARES.  IF THE ANGLE EXCEEDS 30 DEGREES,
C     MULTIPLY FLA BY FNU AND SOLVE NORMAL EQUATIONS AGAIN, AND
C     THEN RECOMPUTE THE RESIDUAL SUM OF SQUARES.
C
      IF(SSQ/PH - 1.0 -EPS1) 450, 450, 420                              MRQT1230
  420 IF(ANGLE -COSGAM) 440, 440, 430                                   MRQT1240
C
C     ANGLE LESS THAN 30 DEGREES. GO BACK AND ADD HALF THE CORRECTION.
C
  430 FK=FK/5.D                                                         MRQT1250
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      WRITE(6,6)I,NSUB,FLA,FK                                           MRQT1260
    6 FORMAT(10X,'I=',I3,3X,'NSUB=',I3,3X,'FLA=',E12.5,3X,'FK=',D12.5)  MRQT1270
      IF(NSUB.EQ.1) FLA=FLA*FNU                                         MRQT1280
      GO TO 380                                                         MRQT1290
C
C     ANGLE GREATER THAN 30 DEGREES . INCREASE FLA AND RESOLVE.
C
  440 FLA = FLA*FNU                                                     MRQT1300
      GO TO 300                                                         MRQT1310
  450 CONTINUE                                                          MRQT1320
C
C     WHEN WE PASS THIS POINT, NEW ESTIMATES HAVE BEEN FOUND WHICH
C     REDUCE THE RESIDUAL SUM OF SQUARES, AND THESE ESTIMATES RESIDE
C     IN THE BI ARRAY.  THE CORRESPONDING RESIDUALS AND RESIDUAL
C     SUM OF SQUARES ARE IN THE ZI ARRAY AND SSQ, RESPECTIVELY.
C
C     NOW DO THE CONVERGENCE TESTS.
C
      DO 460 J = 1,N                                                    MRQT1330
  460 Z(J) = ZI(J)                                                      MRQT1340
C
C     FIRST TEST FOR NO CHANGES IN THE PARAMETERS.
C
      ICON = 0                                                          MRQT1350
      IF (EPS2) 470, 470, 480                                           MRQT1360
  470 KS = 1                                                            MRQT1370
      GO TO 490                                                         MRQT1380
  480 KS = 2                                                            MRQT1390
  490 DO 520 J = 1,K                                                    MRQT1400
      PHL = DABS(BI(J) - B(J))                                          MRQT1410
      B(J) = BI(J)                                                      MRQT1420
      GO TO (520,500), KS                                               MRQT1430
  500 IF(PHL/(1.0E-20 + DABS(B(J))) - EPS2) 520, 510, 510               MRQT1440
  510 ICON = ICON + 1                                                   MRQT1450
  520 CONTINUE                                                          MRQT1460
C
C     IF EPS2 IS GREATER THAN ZERO, THEN ICON WILL CONTAIN THE COUNT
C     OF THE NUMBER OF PARAMETERS WHICH FAIL TO MEET THE
C     CONVERGENCE CRITERION.
C
  530 PHL = DABS(PH - SSQ)                                              MRQT1470
      PH = SSQ                                                          MRQT1480
      IF(EPS1) 551, 551, 540                                            MRQT1490
  540 IF(PHL/PH - EPS1) 551, 550, 550                                   MRQT1500
  550 ICON = ICON + 1                                                   MRQT1510
C
C     IF EPS1 IS GREATER THAN ZERO, THEN ICON WILL BE SET TO 1 IF
C     THE CONVERGENCE REQUIREMENT OF THE RESIDUAL SUM OF SQUARES IS NOT
C     MET.
C
  551 IF(ICON) 720, 720, 555                                            MRQT1520
C
C     IF EPS1 AND EPS2 ARE BOTH NOT ZERO, THEN ICON WILL BE THE SUM OF
C     THE NUMBER OF PARAMETERS NOT MEETING THE CONVERGENCE
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C     REQUIREMENT AND THE FAILURE OF THE RESIDUAL SUM OF SQUARES
C     REQUIREMENT
C
  555 IF(I - NITER) 150, 720, 720                                       MRQT1530
C
C     IF NEITHER CONVERGENCE TEST IS PASSED AND THE NUMBER OF
C     ITERATIONS IS LESS THAN NITER,THE SUBROUTINE RETURNS TO THE
C     CALLING PROGRAM.  IF EITHER/BOTH CONVERGENCE REQUIREMENTS ARE MET,
C     OR THE NUMBER OF ITERATIONS IS 30, WE COMPUTE THE INVERSE OF THE
C     MATRIX OF NORMAL EQUATIONS AND THEN RETURN TO THE CALLING
C     PROGRAM
C
  720 M=0                                                               MRQT1540
      IF(O.EQ.0) RETURN                                                 MRQT1550
      CALL MATINV(QI,K,X,M,DET,LDIMQ)                                   MRQT1560
      DO 721 J = 1,K                                                    MRQT1570
      IF(QI(J,J))724,724,721                                            MRQT1580
  724 WRITE(NO,725)QI(J,J)                                              MRQT1590
  725 FORMAT(/43H NEGATIVE OR ZERO SQUARED ERROR.  QI(J2) = E12.5//     MRQT1600
     X   20X43H THE ERROR MATRIX IS THEREFORE MEANINGLESS./1H )         MRQT1610
      QI(J,J)=1.                                                        MRQT1620
  721 DE(J)=DSQRT(QI(J,J))                                              MRQT1630
      DO 723 J = 1,K                                                    MRQT1640
      DO 723 JJ=1,K                                                     MRQT1650
  723 QI(J,JJ)=QI(J,JJ)/(DE(J)*DE(JJ))                                  MRQT1660
      GO TO 150                                                         MRQT1670
      END                                                               MRQT1680
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80/80 LIST

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12345678901234567890123456789012345678901234567890123456789012345678901234567890

      SUBROUTINE MATINV (A,N,B,M,DET,MA)                                CHMA0010
C                                                                       CHMA0020
C  SOLVES A SYSTEM OF EQUATIONS HAVING A SYMMETRIC POSITIVE-DEFINITE    CHMA0030
C  MATRIX OF COEFFICIENTS.  CALLS CHDEC TO PERFORM THE CHOLESKY         CHMA0040
C  DECOMPOSITION.  CALLS CHSOL TO SOLVE THE SYSTEM.                     CHMA0050
C  THIS MATINV DOES NOT INVERT THE MATRIX A.                            CHMA0060
C  THE INPUT VALUE OF M IS IGNORED.                                     CHMA0070
C  IF THE SOLUTION SUCCEEDS, DET IS RETURNED EQUAL TO 1.0 AND M IS      CHMA0080
C  RETURNED EQUAL TO ZERO.  IF THE SOLUTION FAILS, DET IS RETURNED      CHMA0090
C  EQUAL TO ZERO AND M IS RETURNED EQUAL TO N.                          CHMA0100
C                                                                       CHMA0110
C  J. P. CHANDLER, COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY.   CHMA0120
C                                                                       CHMA0130
      DOUBLE PRECISION A,B,DET
      DIMENSION A(MA,N),B(N)                                            CHMA0140
C                                                                       CHMA0150
      CALL CHDEC (A,N,MA,DET)                                           CHMA0160
      IF(DET)10,10,20                                                   CHMA0170
   10 M=N                                                               CHMA0180
      DET=0.                                                            CHMA0190
      GO TO 30                                                          CHMA0200
   20 CALL CHSOL (A,N,MA,B)                                             CHMA0210
      M=0                                                               CHMA0220
   30 RETURN                                                            CHMA0230
      END                                                               CHMA0240
      SUBROUTINE CHDEC (A,N,MA,DET)                                     CHDE0010
C                                                                       CHDE0020
C  PERFORMS AN IN-PLACE CHOLESKY DECOMPOSITION OF A SYMMETRIC           CHDE0030
C  POSITIVE-DEFINITE MATRIX.  ONLY THE UPPER TRIANGLE OF A IS USED OR   CHDE0040
C  CHANGED.  PIVOTING IS NOT USED.                                      CHDE0050
C                                                                       CHDE0060
C  J. P. CHANDLER, COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY    CHDE0070
C                                                                       CHDE0080
      DOUBLE PRECISION A,DET
      DIMENSION A(MA,N)                                                 CHDE0090
C                                                                       CHDE0100
      ZERO=0.                                                           CHDE0110
      DET=1.                                                            CHDE0120
      DO 100 J=1,N                                                      CHDE0130
      Q=A(J,J)                                                          CHDE0140
      JMU=J-1                                                           CHDE0150
      IF(JMU)30,30,10                                                   CHDE0160
   10 DO 20 K=1,JMU                                                     CHDE0170
   20 Q=Q-A(K,J)**2                                                     CHDE0180
   30 IF(Q)40,40,50                                                     CHDE0190
   40 DET=ZERO                                                          CHDE0200
      GO TO 110                                                         CHDE0210
   50 Q=SQRT(Q)                                                         CHDE0220
      A(J,J)=Q                                                          CHDE0230
      IF(J-N)60,100,100                                                 CHDE0240
   60 JPU=J+1                                                           CHDE0250
      DO 90 K=JPU,N                                                     CHDE0260
      R=A(J,K)                                                          CHDE0270
      IF(JMU)90,90,70                                                   CHDE0280
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80/80 LIST

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12345678901234567890123456789012345678901234567890123456789012345678901234567890

   70 DO 80 L=1,JMU                                                     CHDE0290
   80 R=R-A(K,L)*A(L,J)                                                 CHDE0300
   90 A(J,K)=R/Q                                                        CHDE0310
  100 CONTINUE                                                          CHDE0320
  110 RETURN                                                            CHDE0330
      END                                                               CHDE0340
      SUBROUTINE CHSOL (A,N,MA,B)                                       CHSL0010
C                                                                       CHSL0020
C  SOLVES A SYSTEM OF EQUATIONS HAVING A SYMMETRIC POSITIVE-DEFINITE    CHSL0030
C  MATRIX OF COEFFICIENTS, GIVEN THE CHOLESKY DECOMPOSITION.            CHSL0040
C  ON ENTRY, THE UPPER TRIANGLE OF A CONTAINS THE CHOLESKY-DECOMPOSED   CHSL0050
C  MATRIX AND B CONTAINS THE RIGHT-HAND SIDE.  ON EXIT B CONTAINS THE   CHSL0060
C  SOLUTION VECTOR AND A IS UNCHANGED.                                  CHSL0070
C                                                                       CHSL0080
C  J. P. CHANDLER, COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY    CHSL0090
C                                                                       CHSL0100
      DOUBLE PRECISION A,B
      DIMENSION A(MA,N),B(N)                                            CHSL0110
C                                                                       CHSL0120
C  SOLVE U*Y=B BY FORWARD SUBSTITUTION, WHERE U IS THE UPPER TRIANGULAR CHSL0130
C  CHOLESKY MATRIX.  STORE Y IN B.                                      CHSL0140
C                                                                       CHSL0150
      B(1)=B(1)/A(1,1)                                                  CHSL0160
      IF(N-2)40,10,10                                                   CHSL0170
   10 DO 30 J=2,N                                                       CHSL0180
      SUM=B(J)                                                          CHSL0190
      JMU=J-1                                                           CHSL0200
      DO 20 K=1,JMU                                                     CHSL0210
   20 SUM=SUM-B(K)*A(K,J)                                               CHSL0220
   30 B(J)=SUM/A(J,J)                                                   CHSL0230
C                                SOLVE U*X=Y BY BACK SUBSTITUTION.      CHSL0240
   40 B(N)=B(N)/A(N,N)                                                  CHSL0250
      IF(N-2)80,50,50                                                   CHSL0260
   50 NMU=N-1                                                           CHSL0270
      DO 70 KK=1,NMU                                                    CHSL0280
      K=N-KK                                                            CHSL0290
      SUM=B(K)                                                          CHSL0300
      KPU=K+1                                                           CHSL0310
      DO 60 J=KPU,N                                                     CHSL0320
   60 SUM=SUM-B(J)*A(K,J)                                               CHSL0330
   70 B(K)=SUM/A(K,K)                                                   CHSL0340
   80 RETURN                                                            CHSL0350
      END                                                               CHSL0360
```

VITA

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